

***Dynamical evolution of two associated
galactic bars***

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III Reunión de la REG

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Motivation



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- Double bar galaxies are common in nature
 - Roughly 20% of S0--Sb galaxies are double-barred (rarer in later types)
 - Inner bars ~ 500 pc in radius ($\sim 12\%$ the size of outer bars)
 - sizes range from ~ 100 pc to > 1 kpc (Erwin, 08)
- The evolution and formation of such double-barred galaxies is still not well understood (Debattista & Shen, 07)
- The MW might be double-barred:
 - debate on the detailed morphology of the central region
 - boxy bulge + long thin (in-plane) bar
 - bulge + pseudo bulge
 - elongated boxy bulge
 - ...
- Study case: analytical treatment of the secular evolution of two such bodies, which oscillate with respect each other.
 - without geometrical deformation
 - with deformation described as Jacobi and pseudo-jacobi ellipsoids



$$\vec{\tau}_T = \vec{\tau}_1 + \vec{\tau}_2 = 0 \quad \vec{\tau}_T = \frac{d\vec{L}_T}{dt} \quad \vec{\tau}_i = \frac{d\vec{L}_i}{dt} \quad \vec{L}_i = I_i \vec{\Omega}_i$$

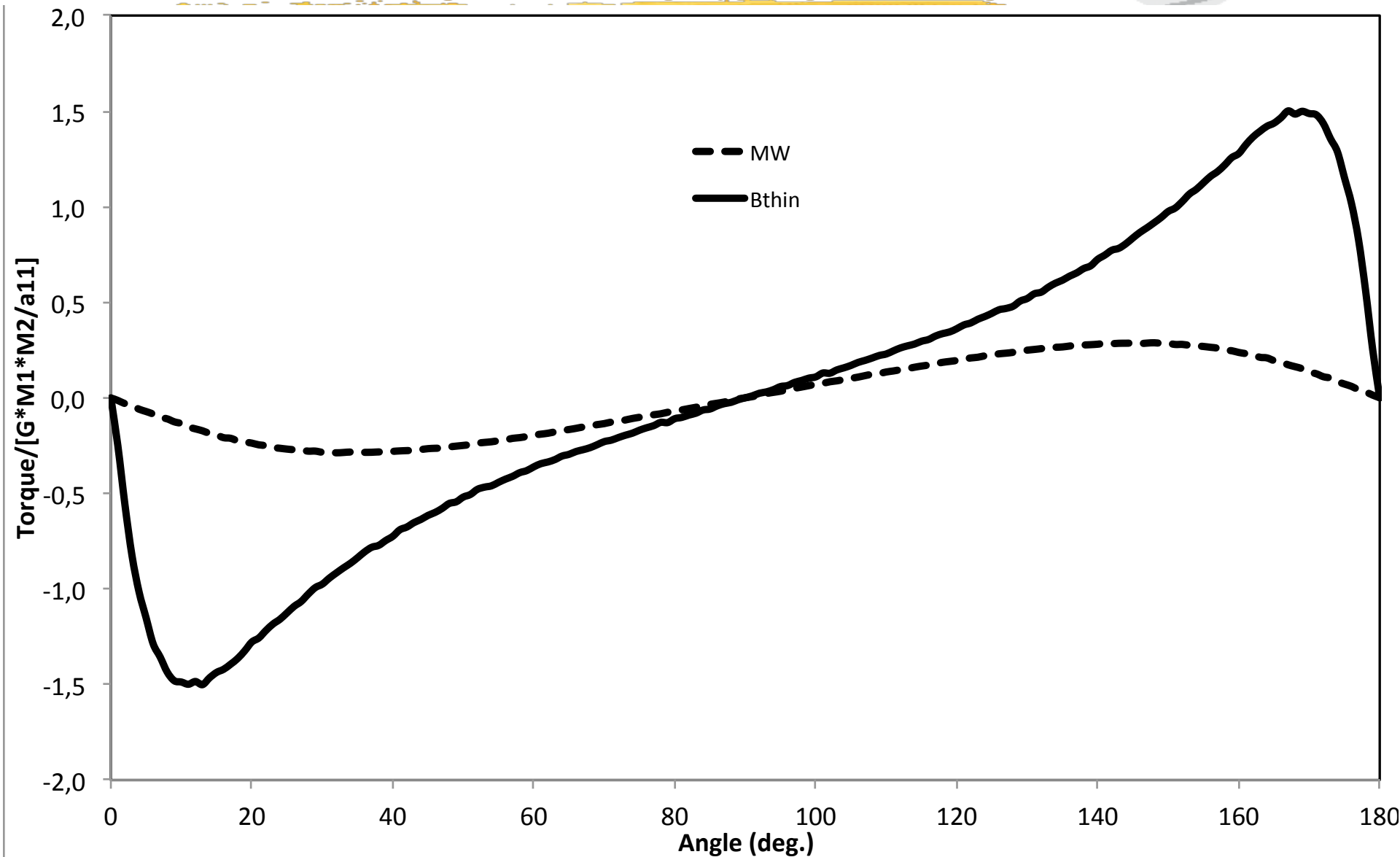
$$\Delta \ddot{\phi} = (\dot{\Omega}_2 - \dot{\Omega}_1) = \left(\frac{1}{I_2} + \frac{1}{I_1} \right) \tau_2(\Delta\phi) - \frac{d \ln(I_2)}{dt} \Omega_2 + \frac{d \ln(I_1)}{dt} \Omega_1$$

$$\tau_j(\Delta\phi) = \int_{V_j} dV \rho_j F_{\phi,i}(\vec{r}) R$$

Torque



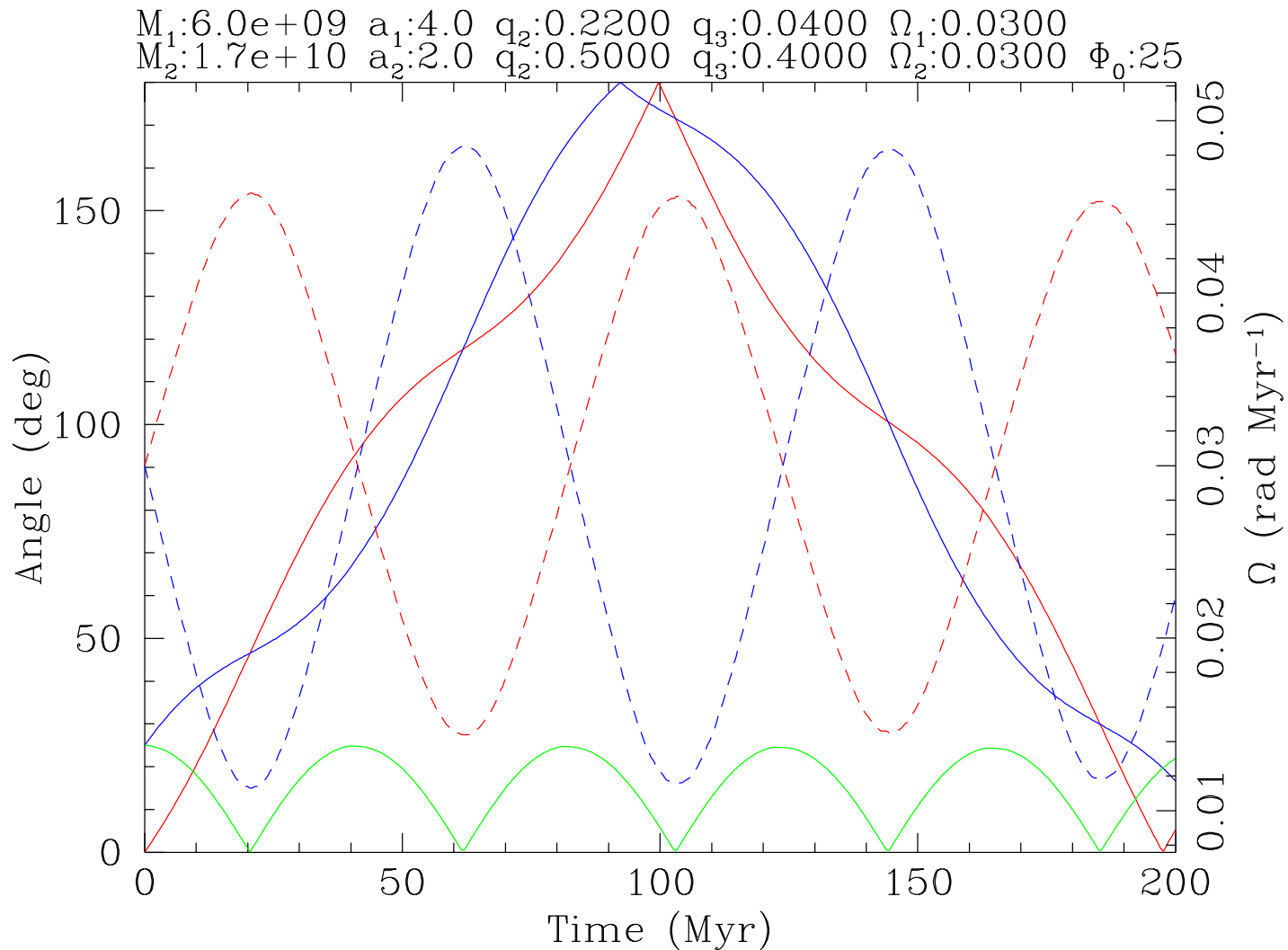
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Rigid Bars



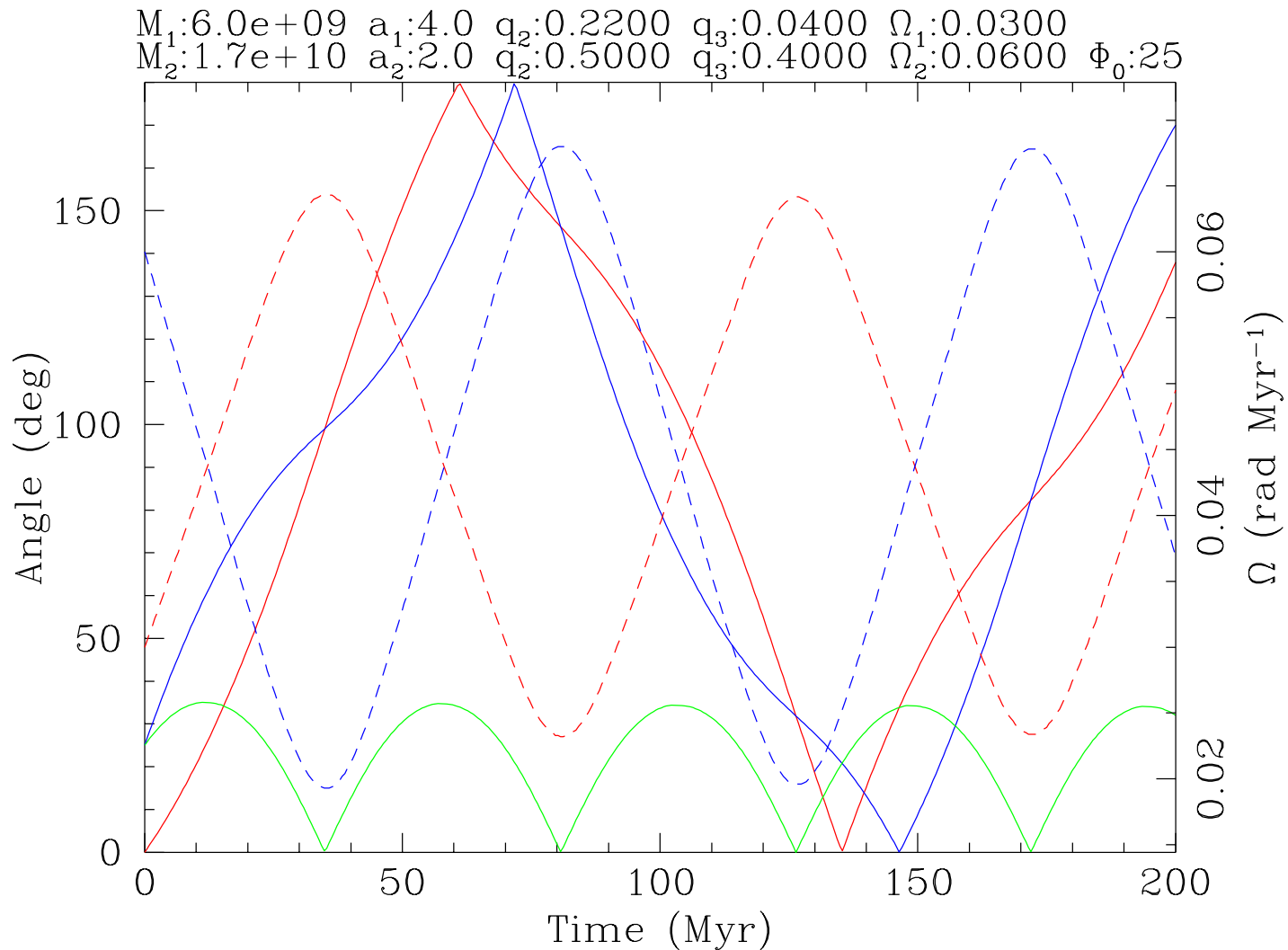
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Rigid Bars



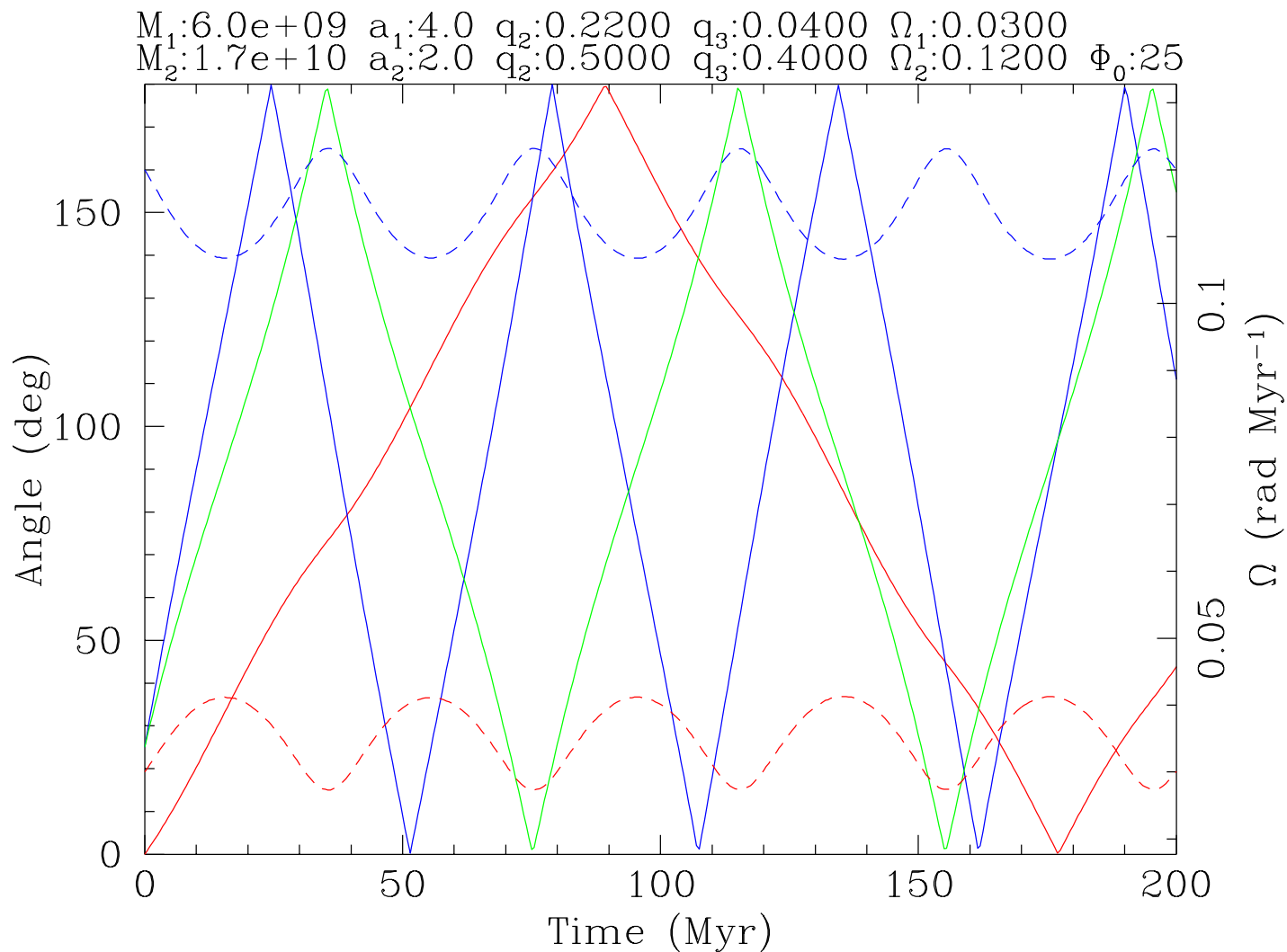
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Rigid Bars



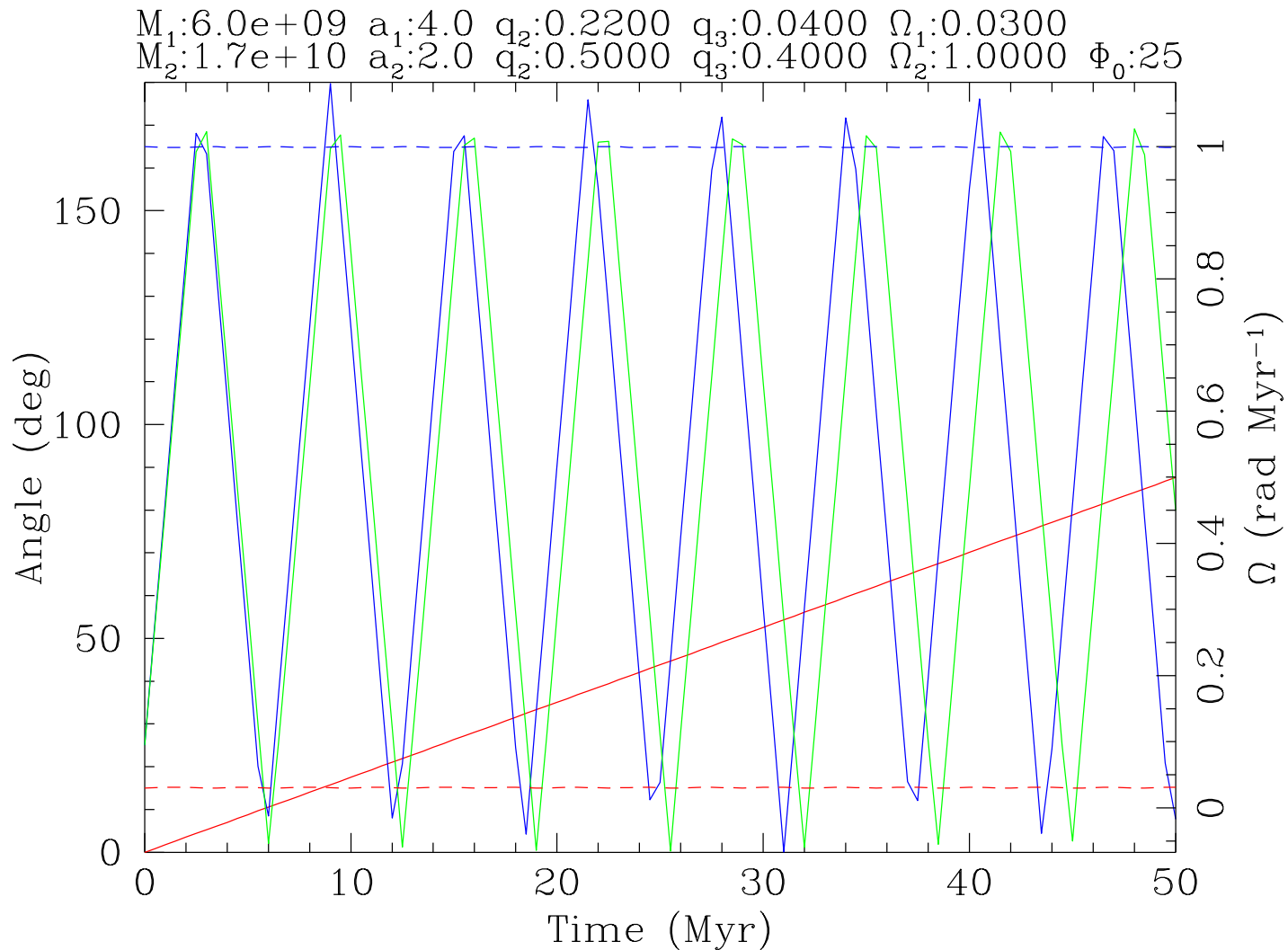
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Rigid Bars



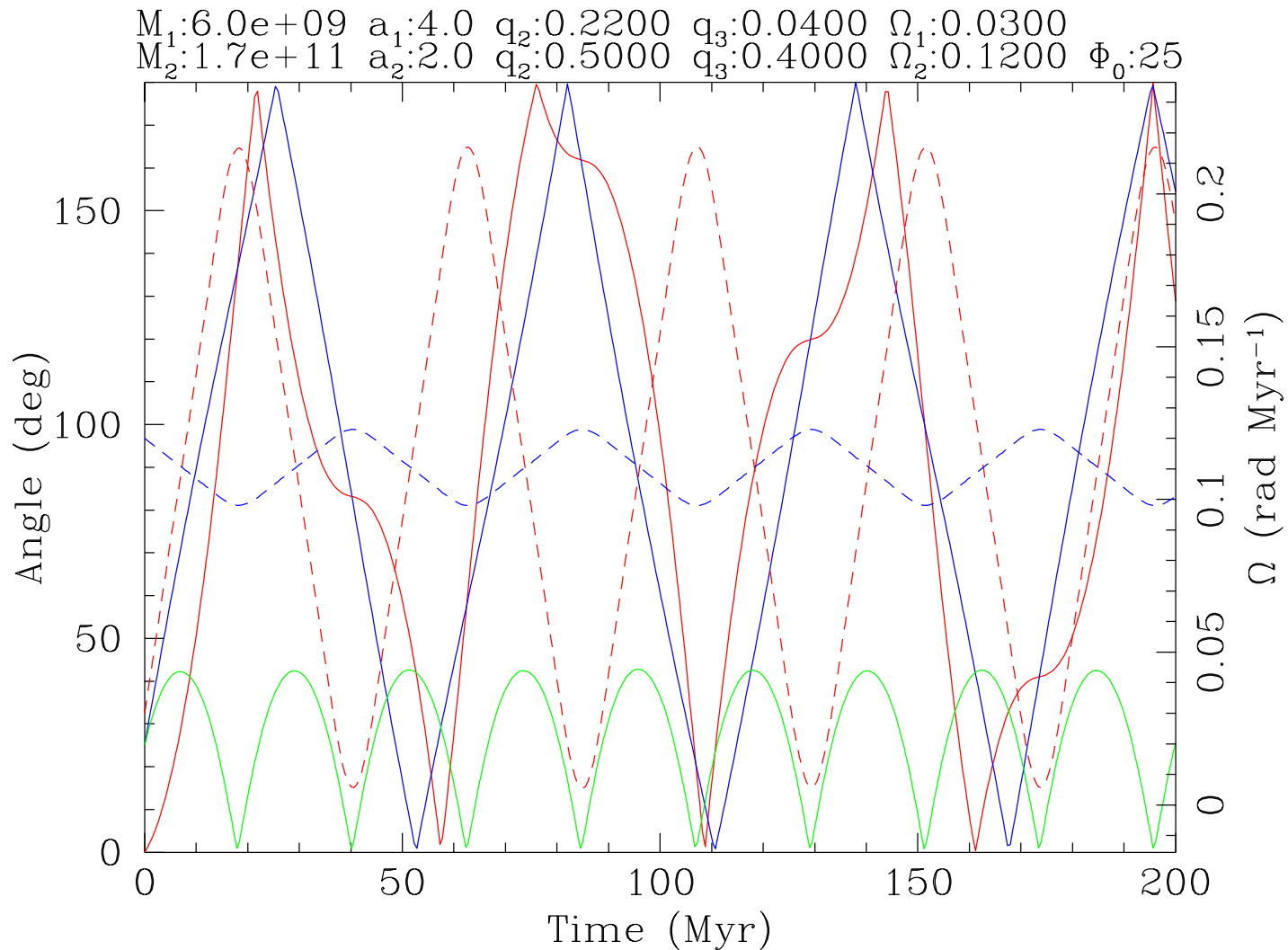
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Rigid Bars



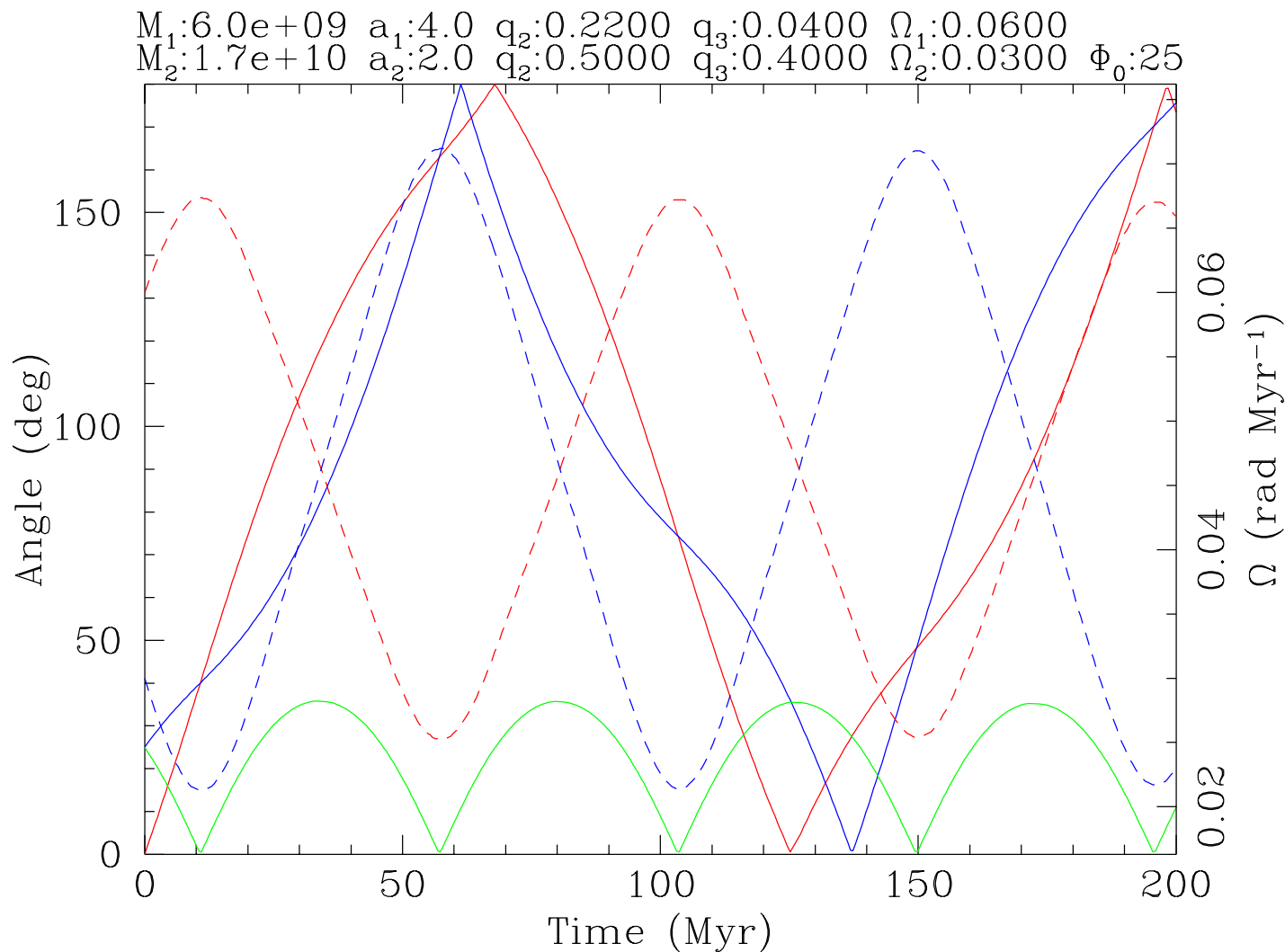
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Rigid Bars



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$$\vec{\tau}_T = \vec{\tau}_1 + \vec{\tau}_2 = 0 \quad \vec{\tau}_T = \frac{d\vec{L}_T}{dt} \quad \vec{\tau}_i = \frac{d\vec{L}_i}{dt} \quad \vec{L}_i = I_i \vec{\Omega}_i$$

$$\Delta \ddot{\phi} = (\dot{\Omega}_2 - \dot{\Omega}_1) = \left(\frac{1}{I_2} + \frac{1}{I_1} \right) \tau_2(\Delta\phi) - \frac{d \ln(I_2)}{dt} \Omega_2 + \frac{d \ln(I_1)}{dt} \Omega_1$$

$$\tau_j(\Delta\phi) = \int_{V_j} dV \rho_j F_{\phi,i}(\vec{r}) R$$

→
$$\dot{\Omega}_i = \tau_i(\Delta\Phi) \left(I_i + \frac{dI_i}{d\Omega_i} \Omega_i \right)^{-1}$$

→
$$\dot{\Omega}_i = \frac{\tau_i(\Delta\Phi)}{I_i} \left(1 + \frac{2\Omega_i}{a_{1i}} \frac{\partial a_{1i}}{\partial \Omega_i} + \frac{2q_{2i}\Omega_i}{1+q_{2i}^2} \frac{\partial q_{2i}}{\partial \Omega_i} \right)^{-1}$$

If Jacobi...



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$$a_2 = a_1 a_3 \left[\frac{A_3}{A_3 a_3^2 - (A_1 - A_2) a_1^2} \right]^{1/2}$$

$$q_2 = q_3 \left[\frac{A_3}{A_3 q_3^2 - (A_1 - A_2)} \right]^{1/2}$$

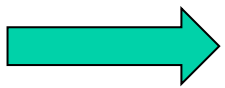
$$a_3 = a_1 a_2 \left[\frac{A_1 - A_2}{A_3 (a_2^2 - a_1^2)} \right]^{1/2}$$

$$q_3 = q_2 \left[\frac{A_1 - A_2}{A_3 (q_2^2 - 1)} \right]^{1/2}$$

$$\Omega^2 = \frac{3}{2} \frac{GM}{a_1 a_2 a_3} \frac{a_1^2 A_1 - a_2^2 A_2}{(a_1^2 - a_2^2)}$$

$$\Omega^2 = \frac{3}{2} \frac{GM}{a_1^3 q_2 q_3} \frac{A_1 - q_2^2 A_2}{(1 - q_2^2)}$$

$$\dot{\Omega}_i = \tau_i (\Delta\Phi) \left[I_i \left(\frac{2q_{2,i} \Omega_i}{1 + q_{2,i}^2} \frac{\partial q_{2,i}}{\partial \Omega_i} - \frac{1}{3} \right) \right]^{-1}$$

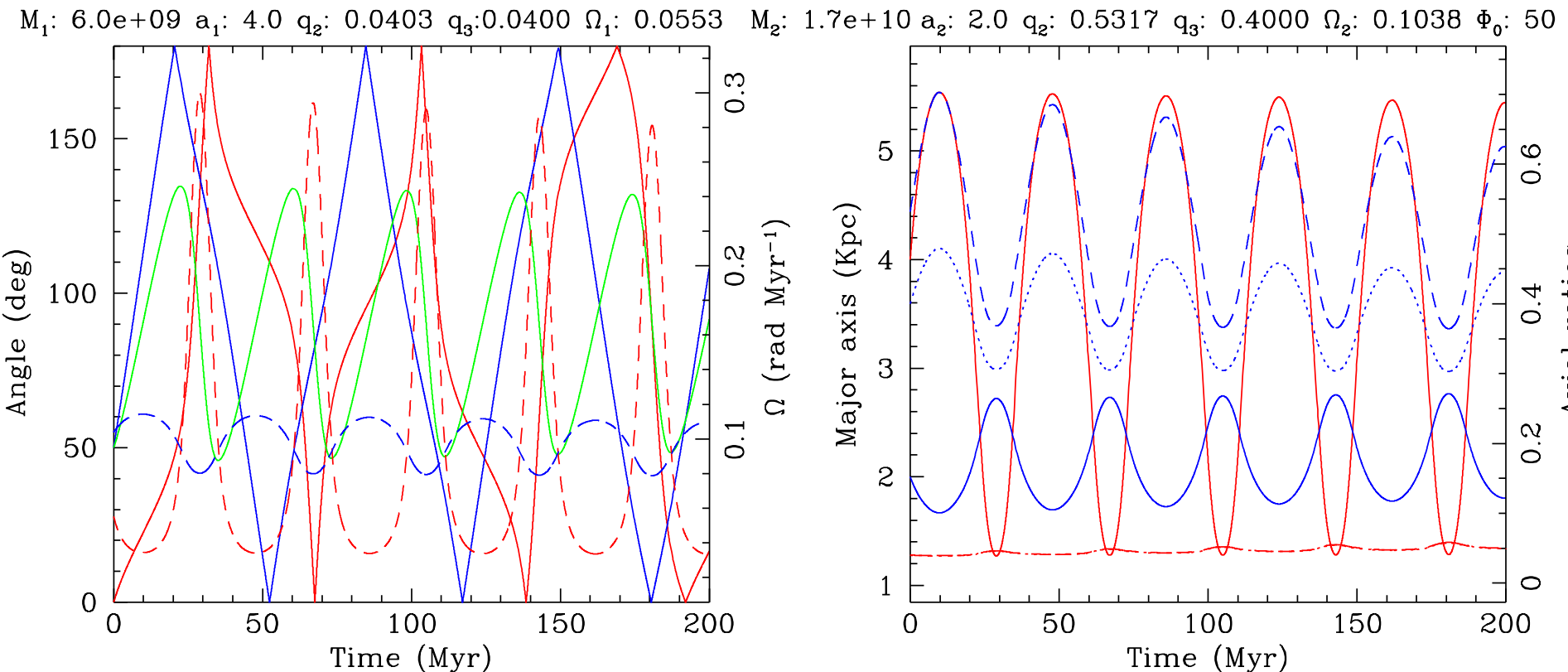


$$\frac{\partial q_{2,i}}{\partial \Omega_i} < 0$$

Evolution of deform. bars (J)



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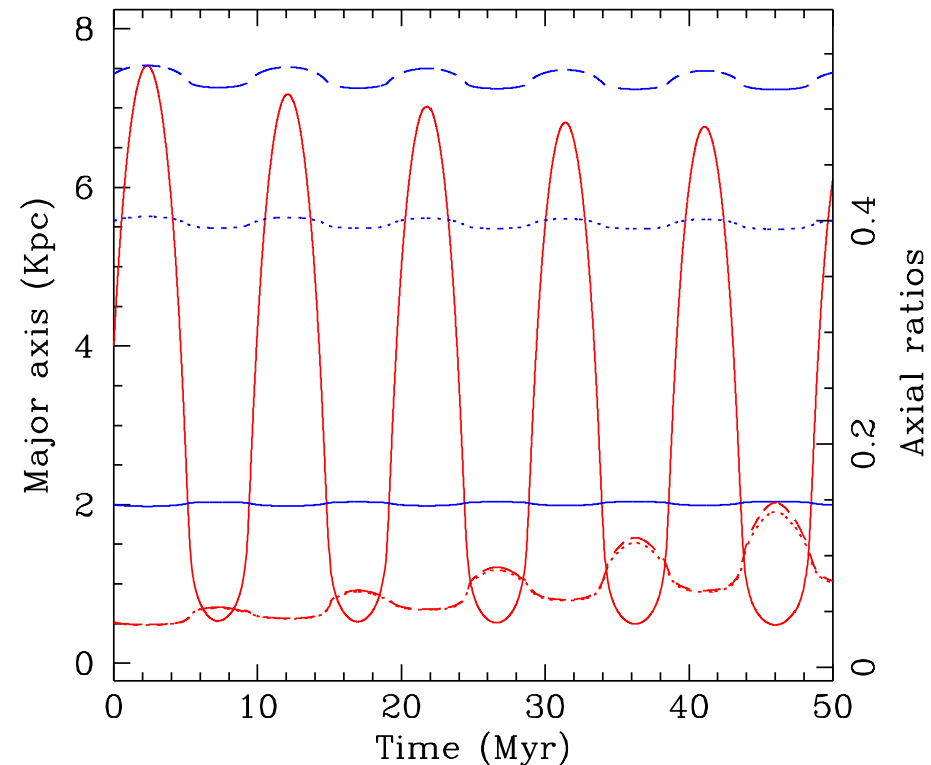
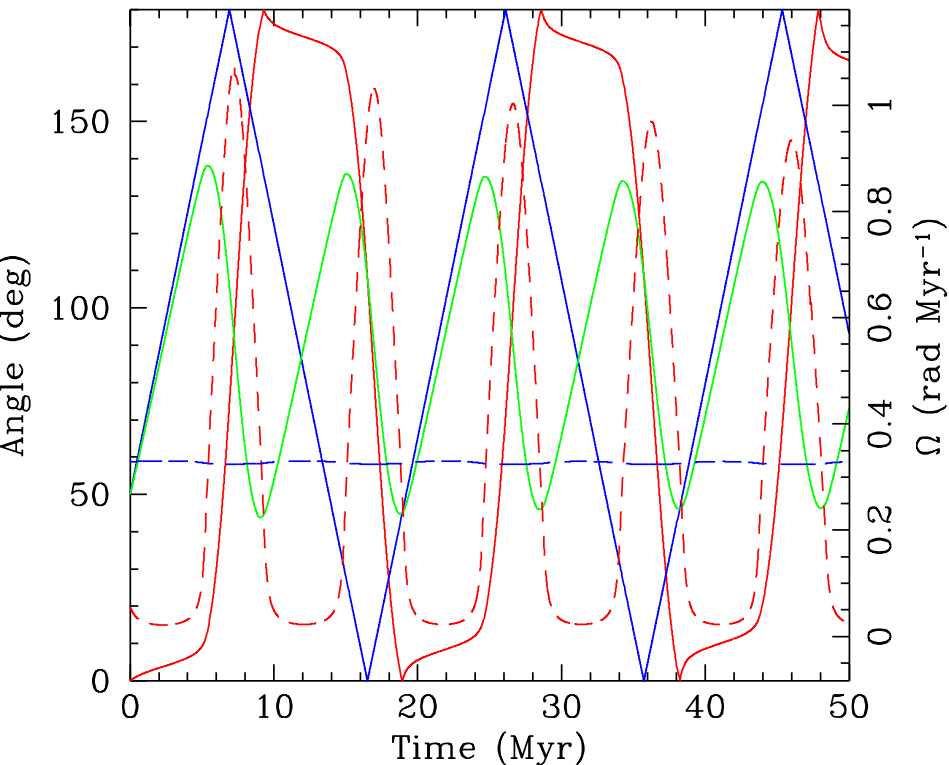


Evolution of deform. bars (J)



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$M_1: 6.0e+09$ $a_1: 4.0$ $q_2: 0.0403$ $q_3: 0.0400$ $\Omega_1: 0.0553$ $M_2: 1.7e+11$ $a_2: 2.0$ $q_2: 0.5317$ $q_3: 0.4000$ $\Omega_2: 0.3282$ $\Phi_0: 50$

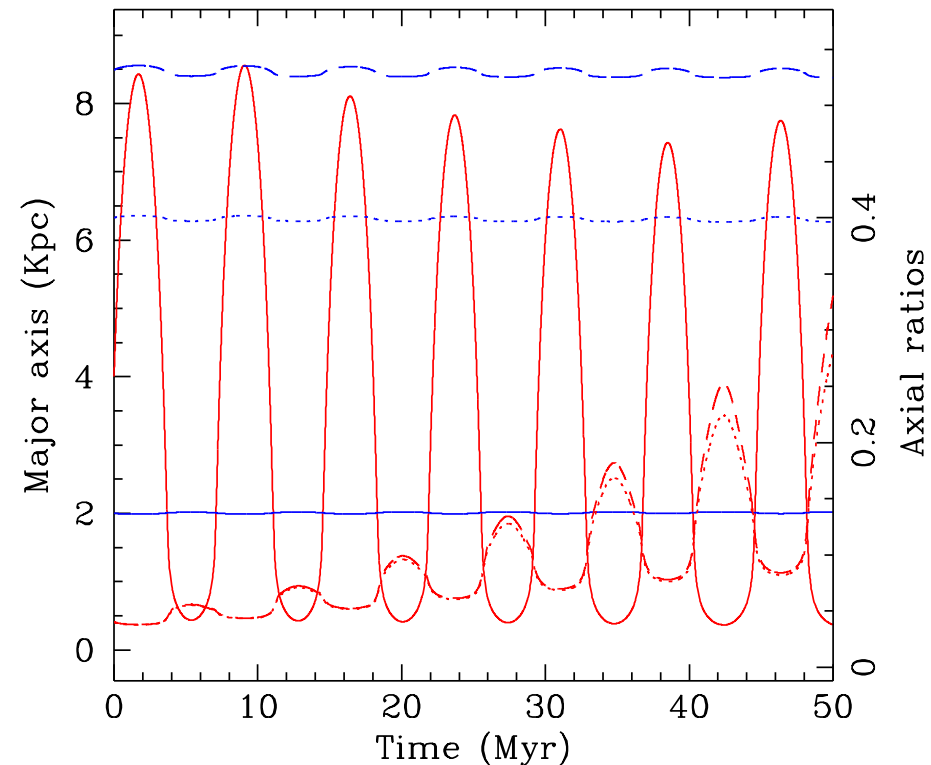
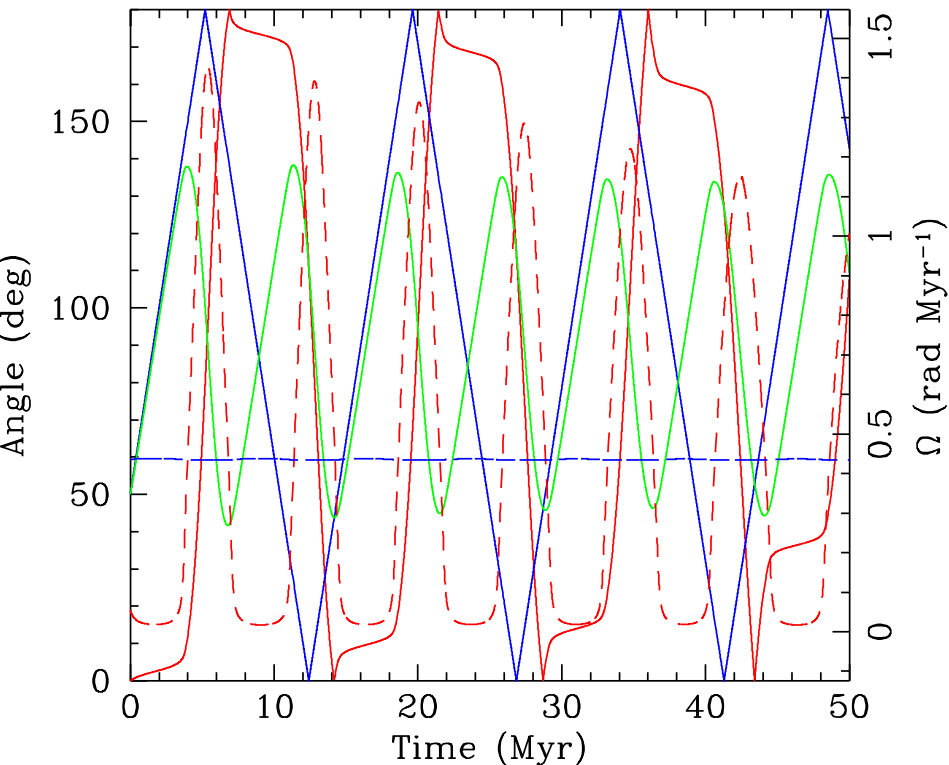


Evolution of deform. bars (J)



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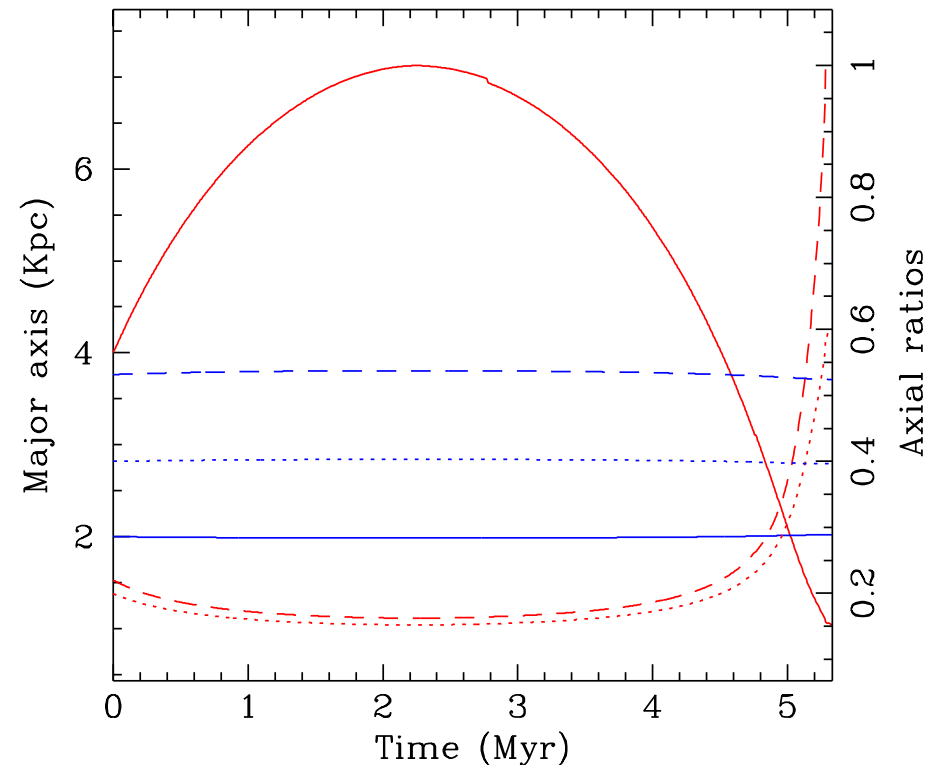
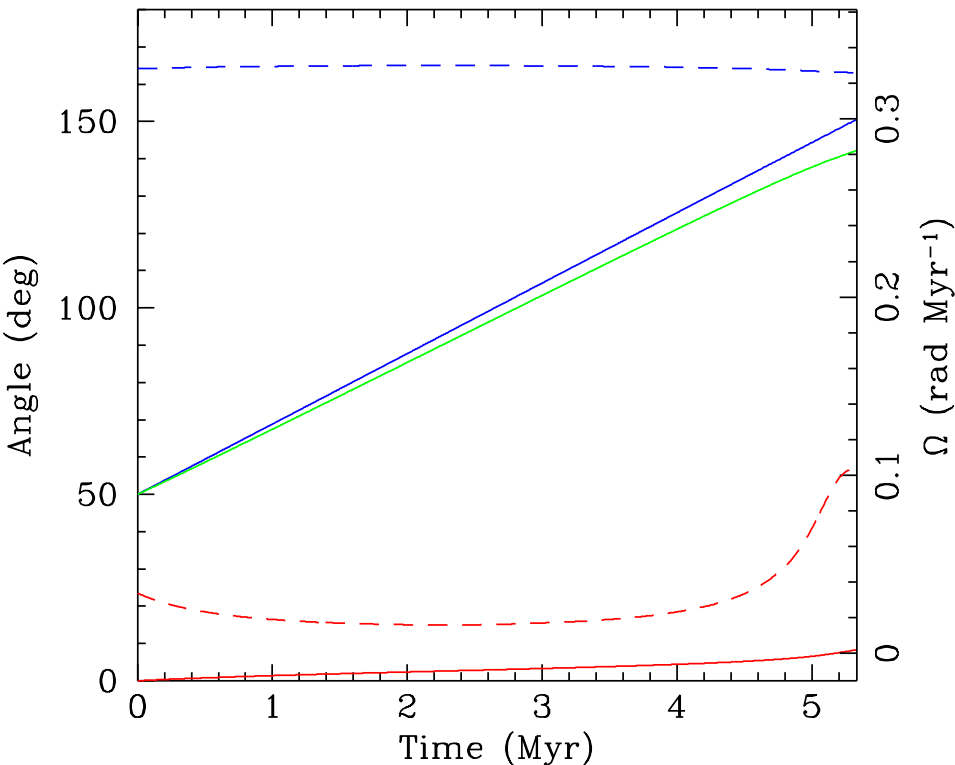


Evolution of deform. bars (J)



gaia

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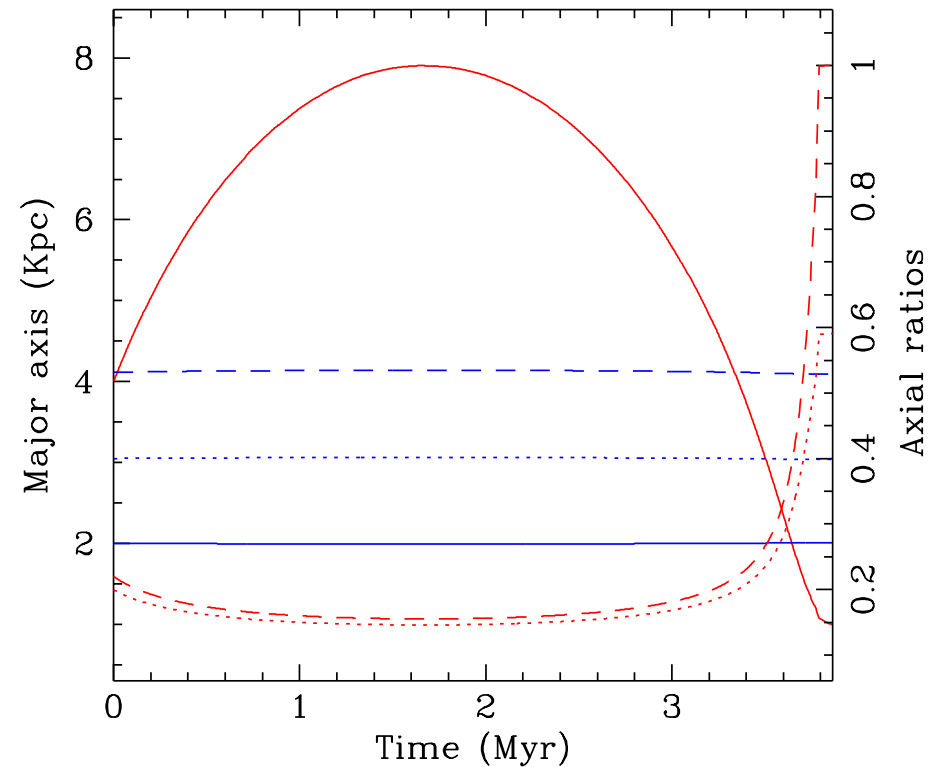
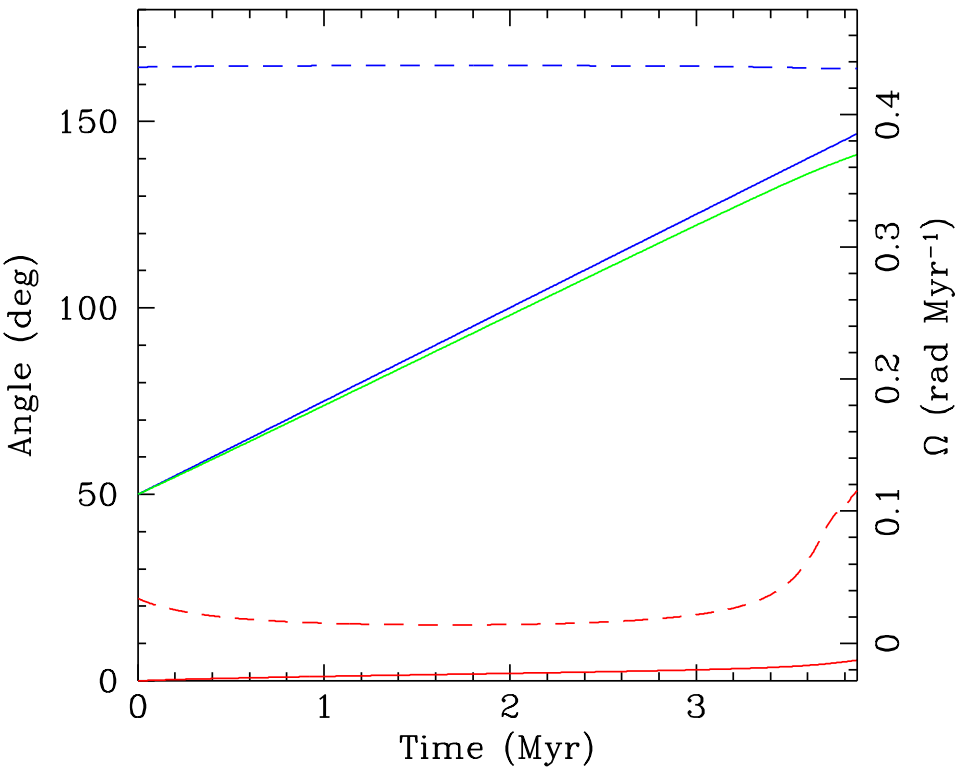


Evolution of deform. bars (J)



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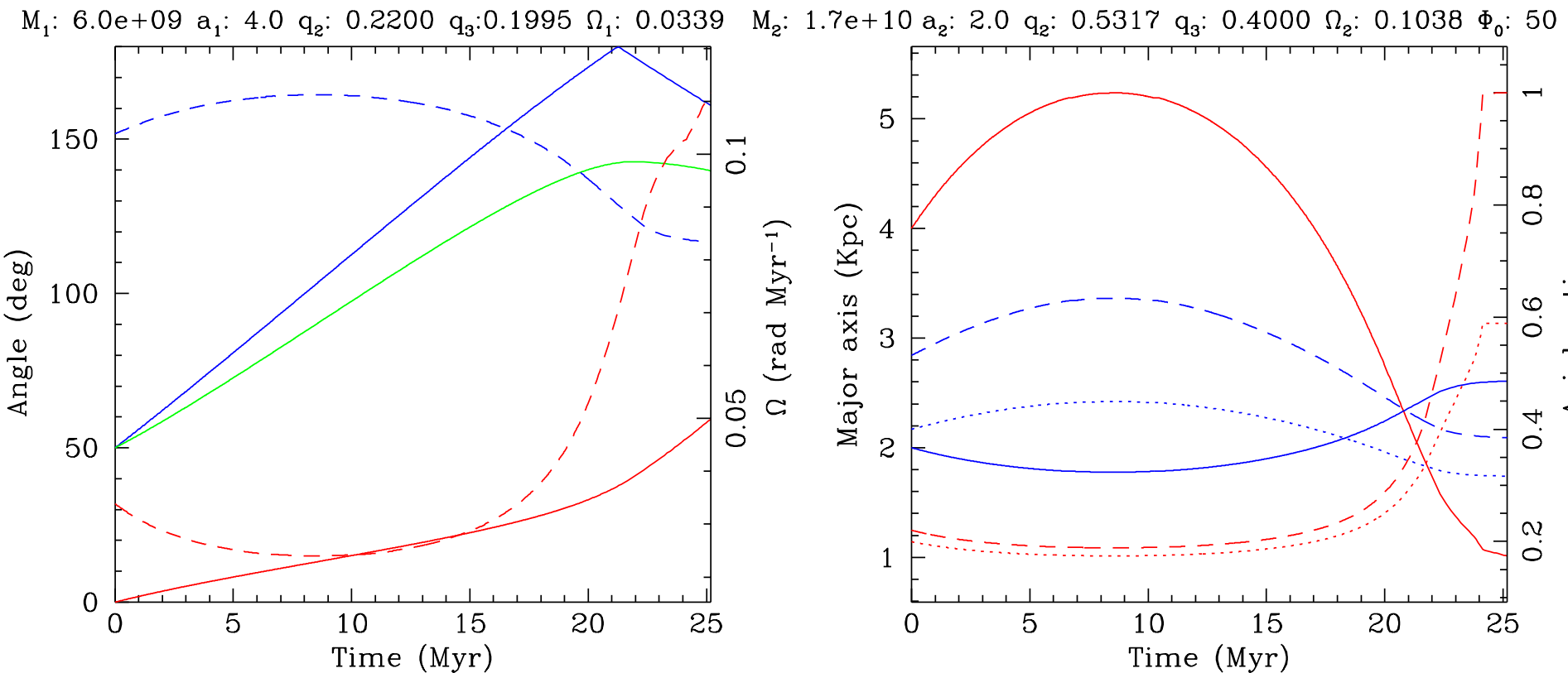
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Evolution of deform. bars (J)



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If pseudo-Jacobi



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$$V(\vec{x}) = \frac{3}{4} \frac{GM}{a_1^3} \int_{u(\vec{x})}^{\infty} \left[1 - \sum_{j=1}^3 \frac{x_j^2}{a_j^2 + \mu} \right] \frac{d\mu}{\Delta(\mu)}$$

$$\Delta^2(\mu) = (1 + \mu)(q_2^2 + \mu)(q_3^2 + \mu) \quad \sum_{j=1}^3 \frac{x_j^2}{a_j^2 + u} = 1$$

$$V_i(\vec{x}) = -\frac{3}{4} \frac{GM_i}{a_{1i}^3} \left[A_i[u_i(\vec{x})]x^2 + B_i[u_i(\vec{x})]y^2 + C_i[u_i(\vec{x})]z^2 \right]$$

$$A_i[u_i(\vec{x})] = \int_{u_i(\vec{x})}^{\infty} \frac{d\mu}{(1 + \mu)\Delta_i(\mu)}$$

$$B_i[u_i(\vec{x})] = \int_{u_i(\vec{x})}^{\infty} \frac{d\mu}{(q_2^2 + \mu)\Delta_i(\mu)}$$

$$C_i[u_i(\vec{x})] = \int_{u_i(\vec{x})}^{\infty} \frac{d\mu}{(q_3^2 + \mu)\Delta_i(\mu)}$$

and...



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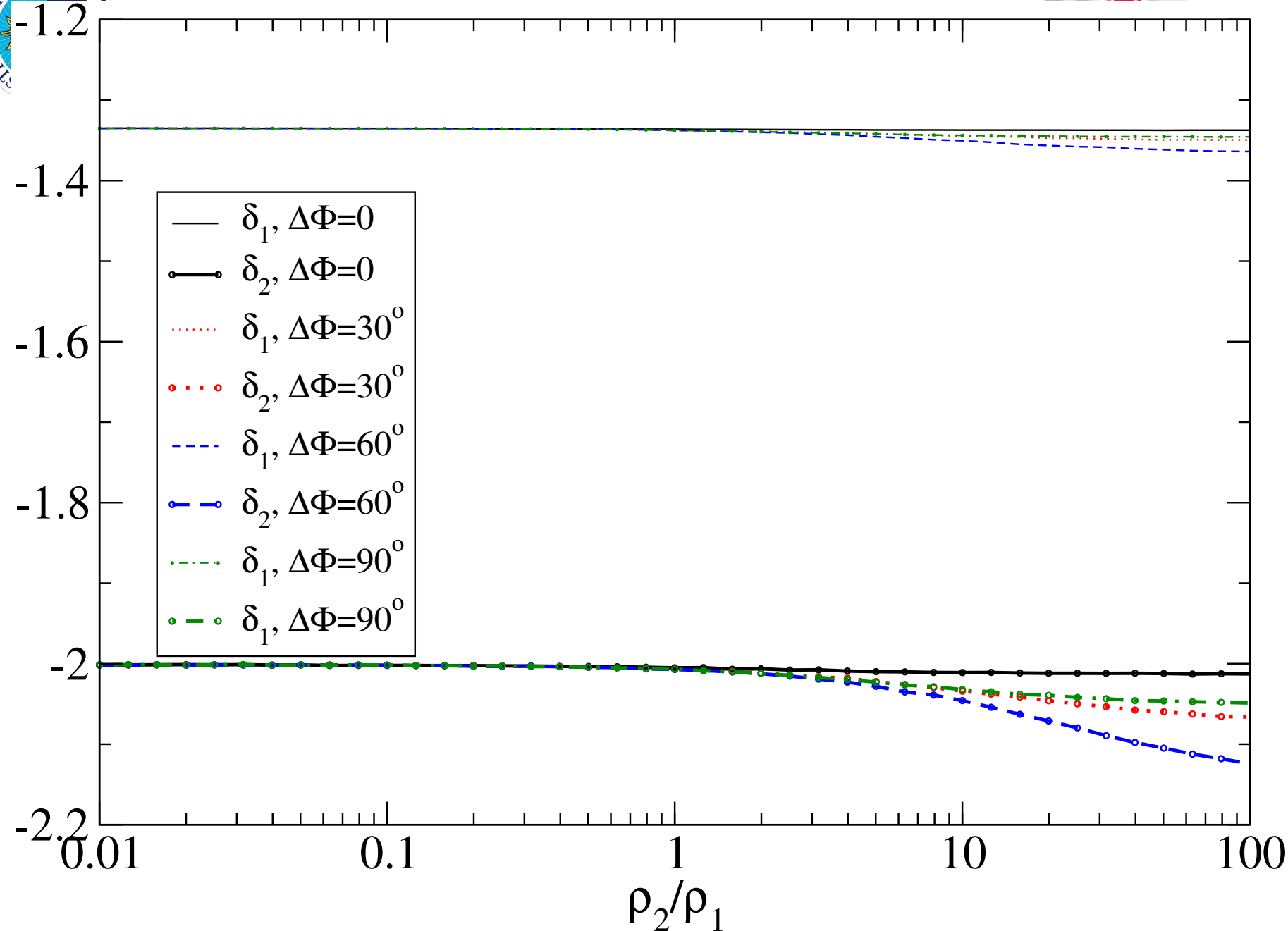
$$V_i(\vec{x}_i) + V_j(\vec{x}_i) + \frac{1}{2}\Omega_i^2(x_i^2 + y_i^2) = \text{constant}$$

$$\begin{aligned} A_i[0] + D_{ji} \left[A_j[u_{ij1}] \cos^2(\Delta\Phi) + B_j[u_{ij1}] \sin^2(\Delta\Phi) \right] - E_i &= \\ = q_{2i}^2 \left\{ B_i[0] + D_{ji} \left[A_j[u_{ij2}] \sin^2(\Delta\Phi) + B_j[u_{ij2}] \cos^2(\Delta\Phi) \right] - E_i \right\} &= \\ = q_{3i}^2 \left\{ C_i[0] + D_{ji} C_j[u_{ij3}] \right\} \end{aligned}$$

$$\dot{\Omega}_i = \frac{\tau_i(\Delta\Phi)}{I_i} \left(1 + \frac{2\Omega_i}{a_{1i}} \frac{\partial a_{1i}}{\partial \Omega_i} + \frac{2q_{2i}\Omega_i}{1+q_{2i}^2} \frac{\partial q_{2i}}{\partial \Omega_i} \right)^{-1}$$

$$\delta_1 \equiv (2\Omega_i / a_{1i}) \times (\partial a_{1i} / \partial \Omega_i)$$

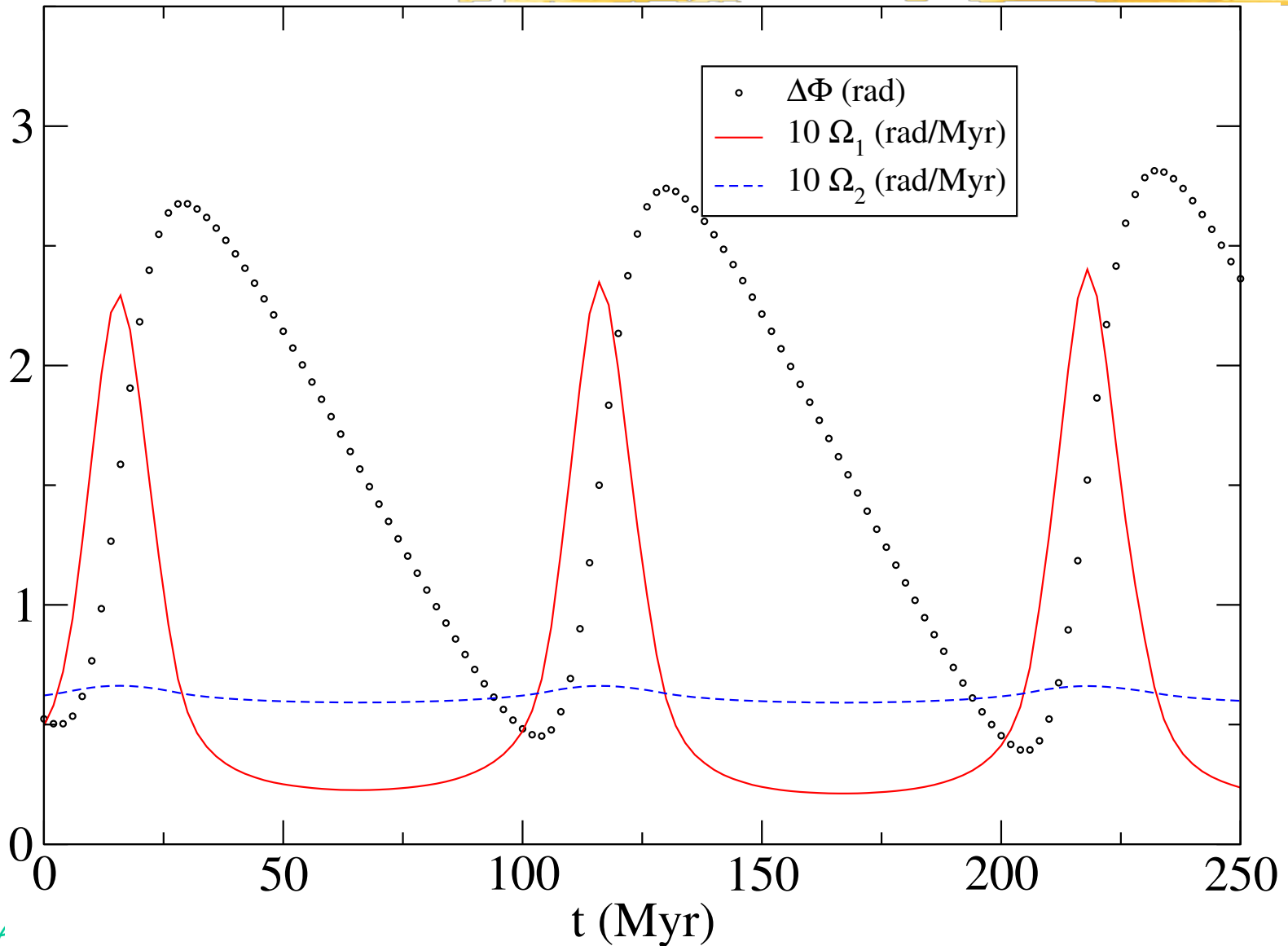
$$\delta_2 \equiv \Omega_i \times (\partial q_{2i} / \partial \Omega_i)$$



Evol. of deform. bars (PJ)



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Evol. of deform. bars (PJ)



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