Internal Structure of Stellar Clusters: Geometry of Star Formation

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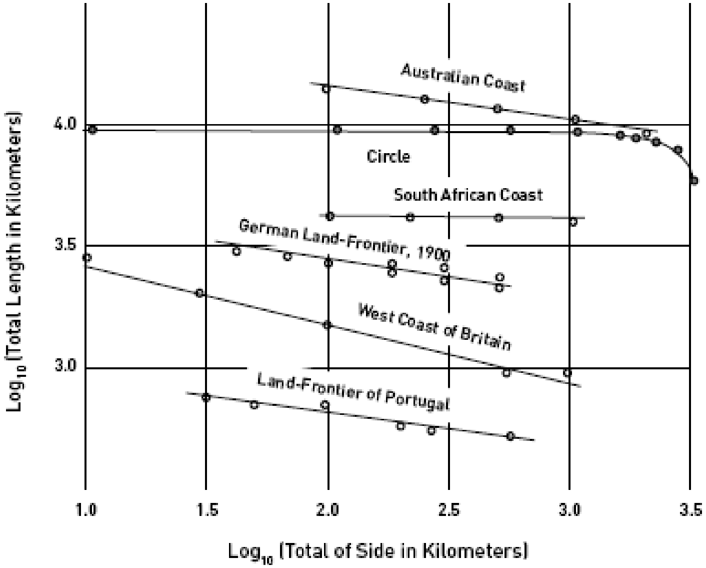
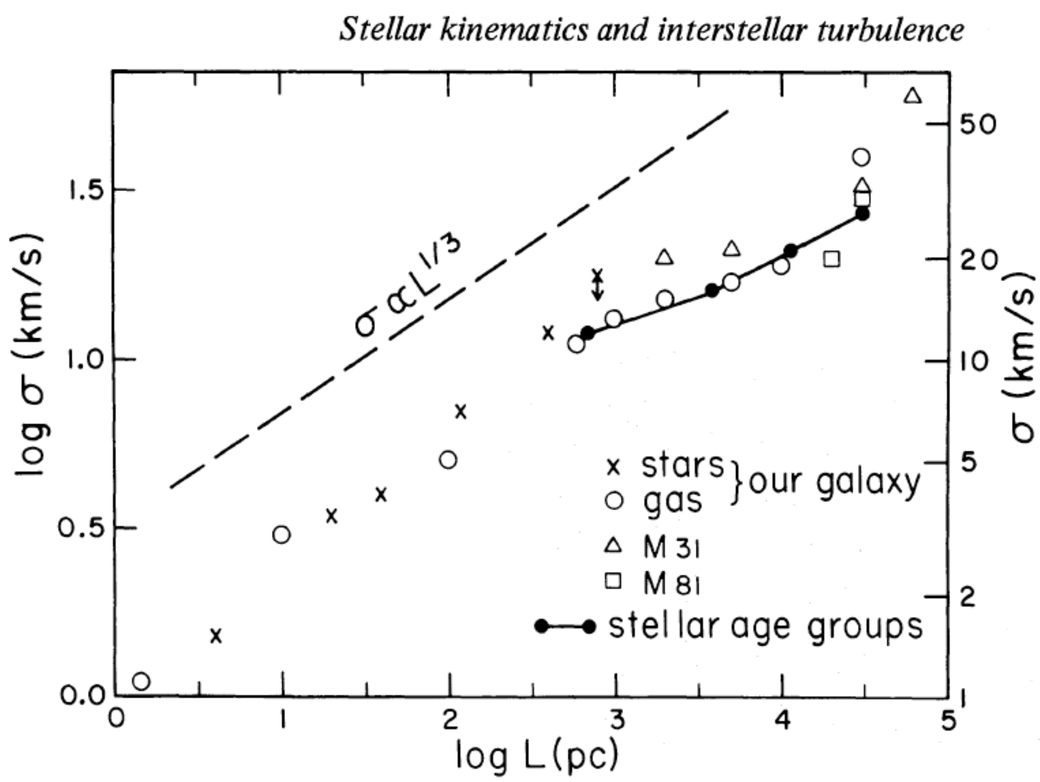
Fractal Geometry

Mandelbrot, B. 1967, *How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension.* *Science, New Series, Vol. 156, No. 3775. (May 5, 1967), pp. 636-638*



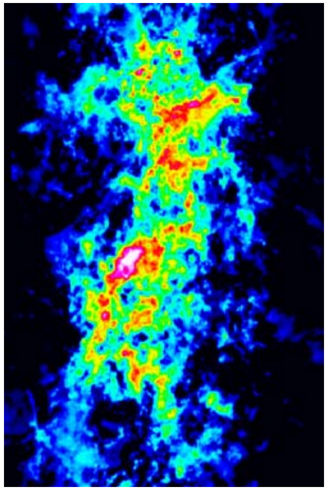
Opening a new geometrical description of Nature

Fractal systems characterize by some physical properties scaling with spatial length as a power function



(Larson 1979)

Connection between Geometry and Physics of the Star Formation

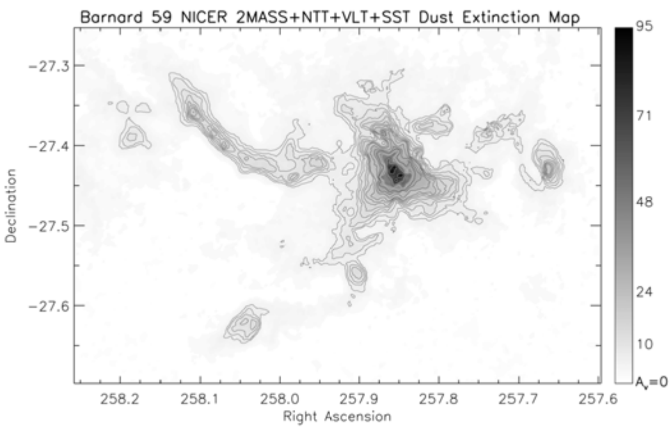


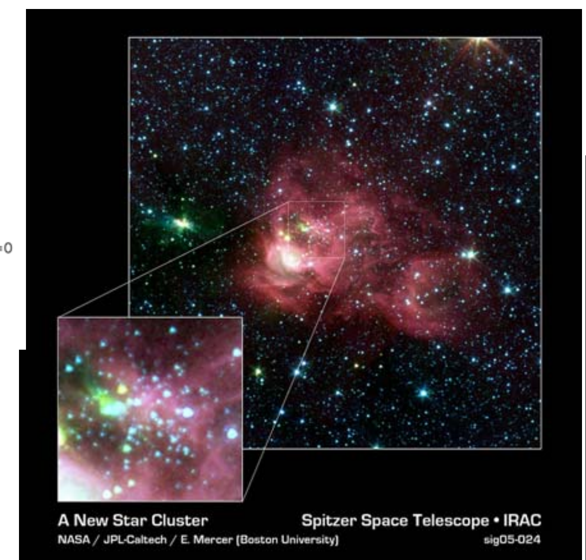
Clusters

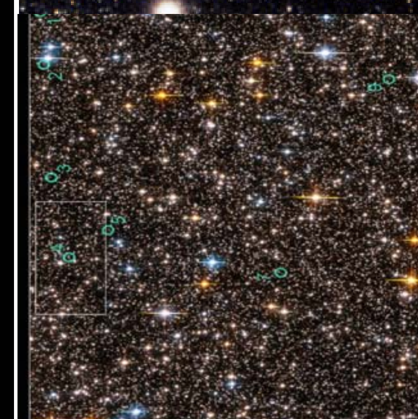
- Radial Distribution
- Clumpy Structure
- Remnant of Diluted Cluster

Molecular Clouds

Pre-stellar Cores







Embedded Clusters

How to approach the program?

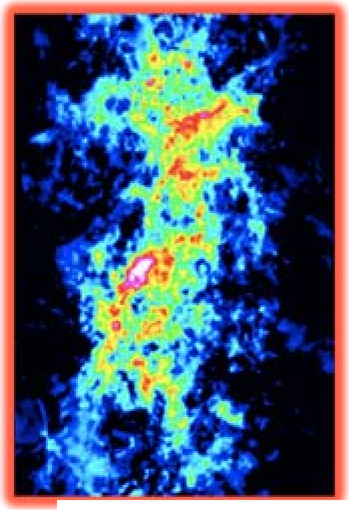
Models & Observations

- Models:
- Observations:

- We need to know the complete phase space of the different cluster stages (spatial position & kinematic structure); [a lack of precise proper motion and radial velocity data, so far].
- Photometric data providing individual SEDs and hence information on the current evolutionary state of the system; [enlarge the wavelength coverage for large databases].
- Objective and homogeneous tools for describing the current state of the stellar systems and for comparing different observational data with model predictions.

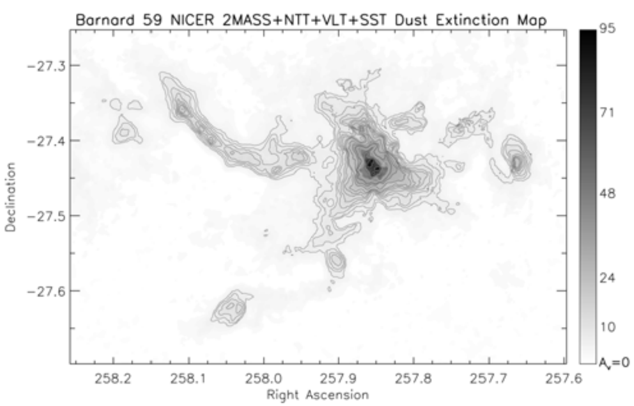
Clusters

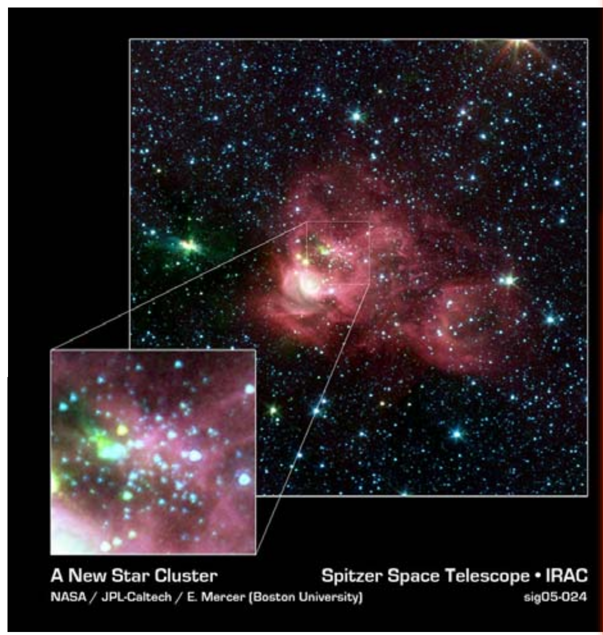
- Radial Distribution
- Clumpy Structure
- Remnant of Diluted Cluster



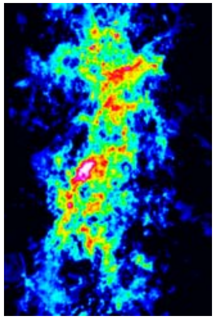
Molecular Clouds

Pre-stellar Cores



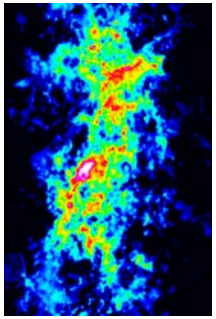


Embedded Clusters



Questions:

- Assuming they show a fractal structure. Is there a universal fractal dimension for molecular clouds?
- If variable, what is the nature and scale of the variation?
    - Constant within a galaxy?
    - Dependent on the local environment?
- Does the fractal dimension derive from the physical processes driving gas turbulence?
    - If true, is that relation unique, or could it be a convolved result of several physical mechanisms?



Answers (or so) :

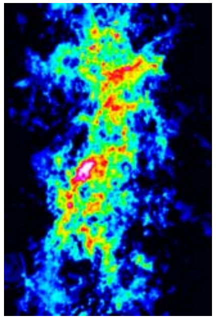
What is the best method to get the Hausdorff dimension (H) of emission maps?

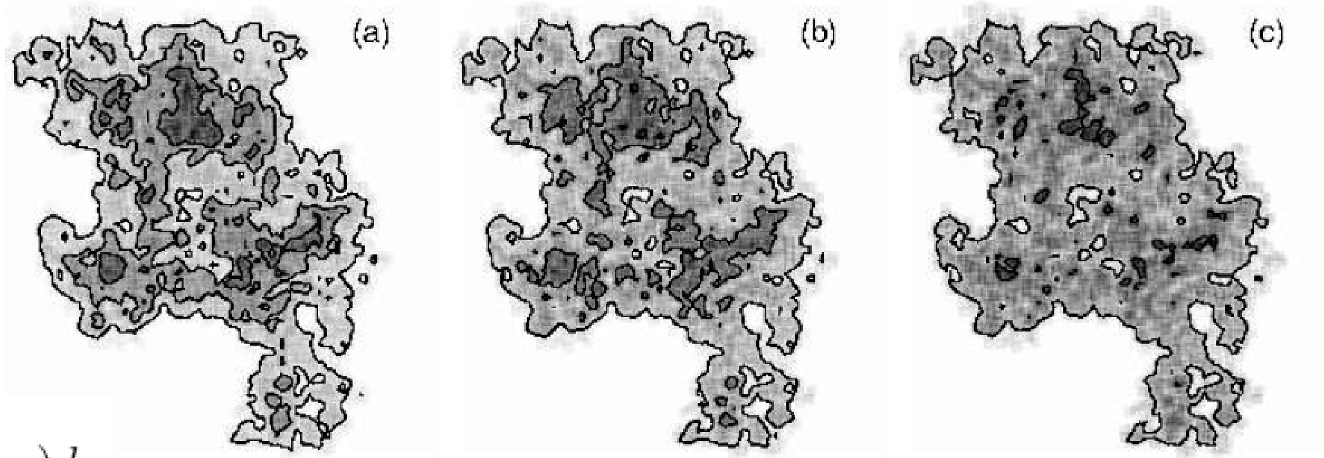
$$A \sim L^H$$

- Correlation Function (Grassberger & Procaccia 1983; Minkowski 1900)
- Perimeter-Area (Mandelbrot 1983)
- Box Counting (Mandelbrot 1983)
- Mass Dimension (Mandelbrot 1983)
- Q parameter adapted to emission maps (Cartwright, Whitworth & Nutter 2006)

Taking into account and minimizing effects due to:

- Opacity
- Noise
- Spatial Resolution
- Projection





$$\tau(x,y) = c\int_{z_0}^{z} \rho(x,y,z)dz$$

$$\tau_0 = 3c/(2\pi)$$

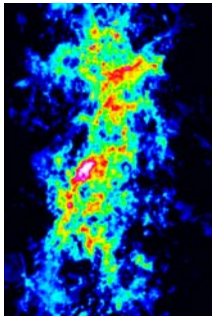
Fig. 1.— Three images projected of the same cloud with fractal dimension $D_f = 2.6$ but for three different optical depth values: (a) $\tau_0 = 0$, (b) $\tau_0 = 1$ and (c) $\tau_0 = 2$. The contour levels are fixed at 25%, 50% and 75% of the maximum projected intensity for the case $\tau_0 = 0$.

Table 1. Calculated fractal dimension

Perimeter-area based dimension (D_{per})				
D_f	$\tau_0 = 0.0$	$\tau_0 = 1.0$	$\tau_0 = 2.0$	$\tau_0 = 5.0$
2.0	1.601±0.024	1.602±0.021	1.604±0.021	1.591±0.019
2.3	1.469±0.023	1.474±0.021	1.467±0.019	1.455±0.018
2.6	1.359±0.032	1.364±0.032	1.367±0.035	1.359±0.046
Mass dimension (D_m)				
D_f	$\tau_0 = 0.0$	$\tau_0 = 0.5$	$\tau_0 = 1.0$	$\tau_0 = 1.25$
2.6	1.808±0.029	1.816±0.034	1.876±0.045	1.860±0.044

(Sánchez, Alfaro, & Pérez 2007)

Opacity



Noise

Noise biases the estimation of D_{per} (Vogelar & Wakker 1994; Lee 2004; Sánchez et al. 2007).

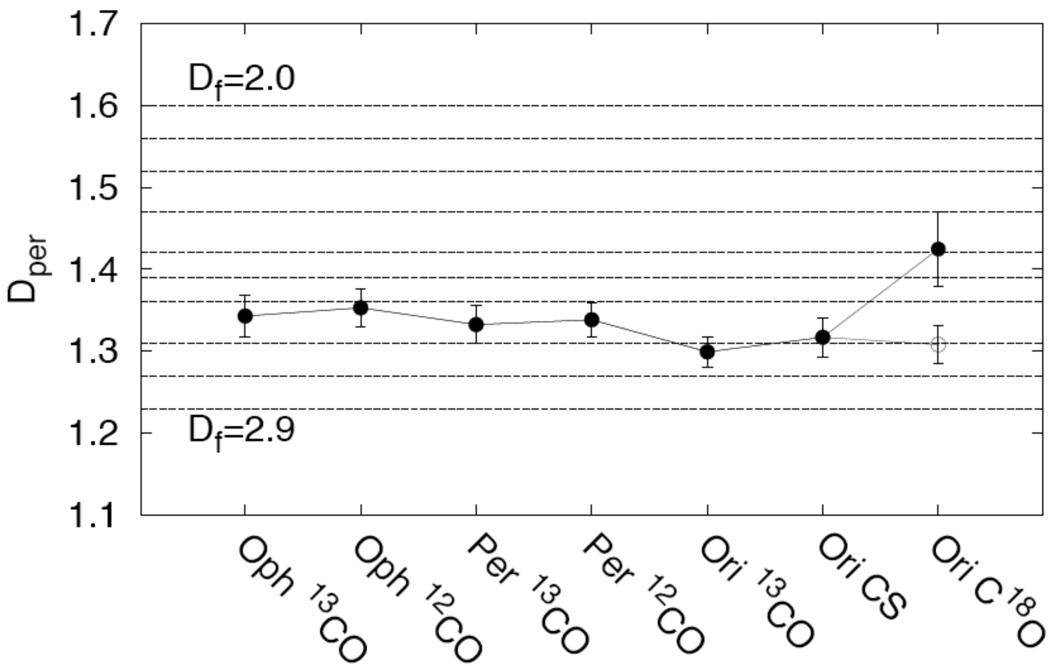
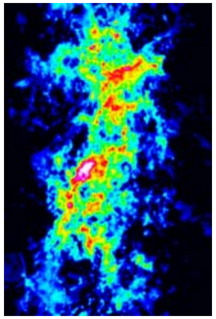


Fig. 3.— The perimeter dimension D_{per} obtained for each molecular cloud map. The dashed horizontal lines indicate values calculated in Paper I for fractal dimension values D_f from 2.0 to 2.9 in increments of 0.1. The open circle refers to the result obtained for the smoothed $C^{18}O$ map (see text).

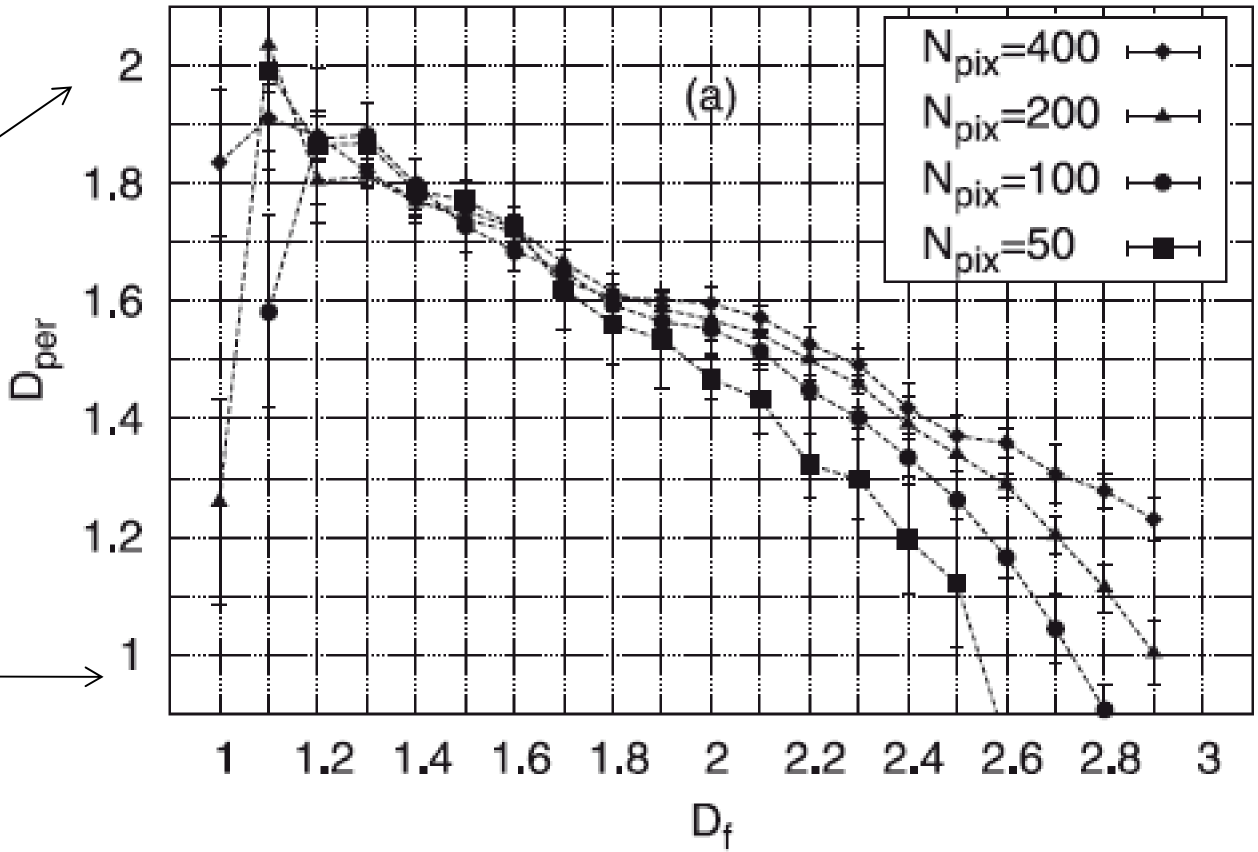


Projection & Resolution

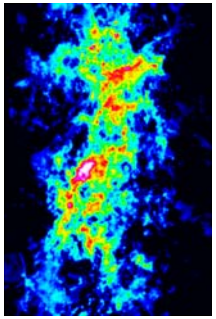
Filamentary

$$P = A^{(D_{per}/2)}$$

Homogeneous



(Sánchez, Alfaro & Pérez 2005)



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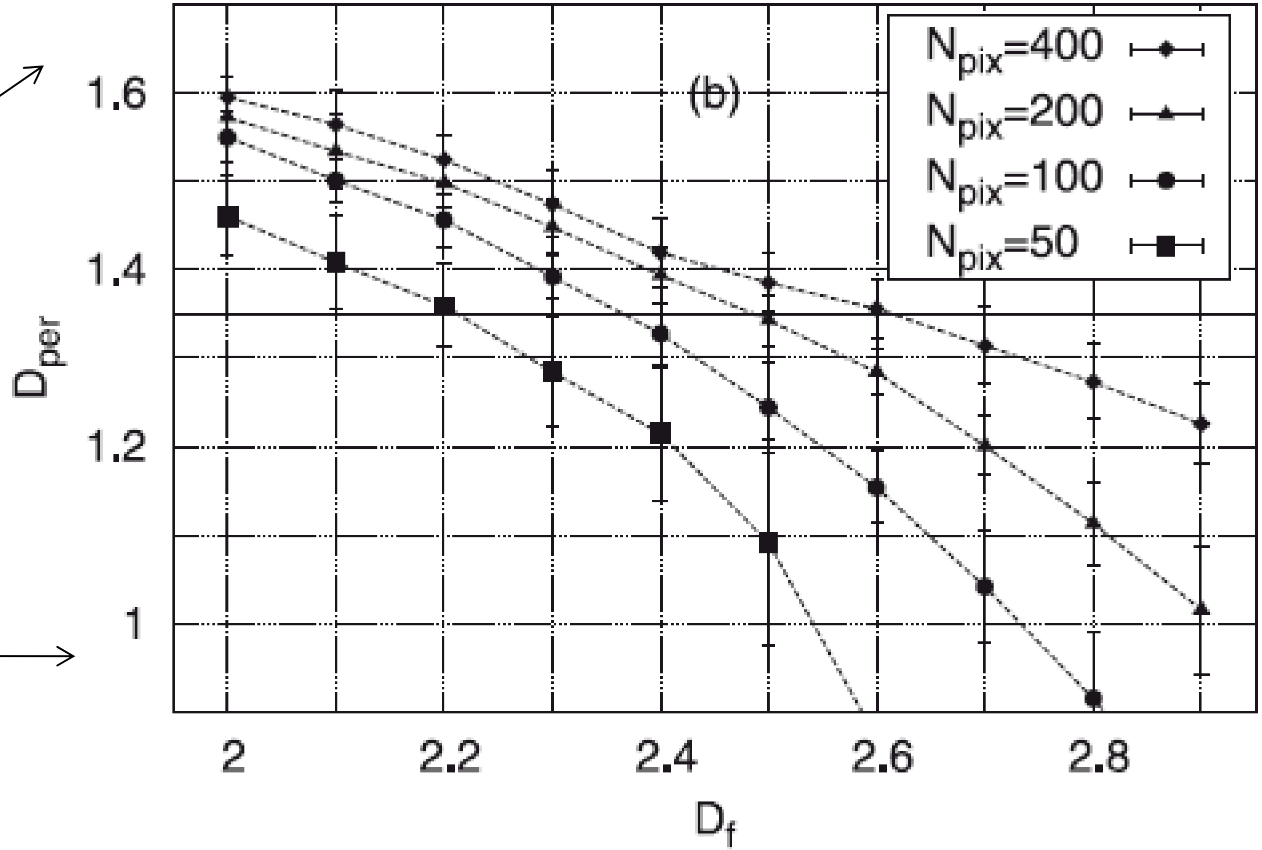
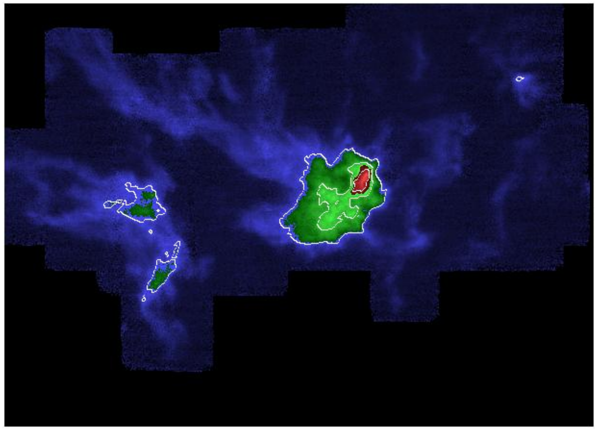
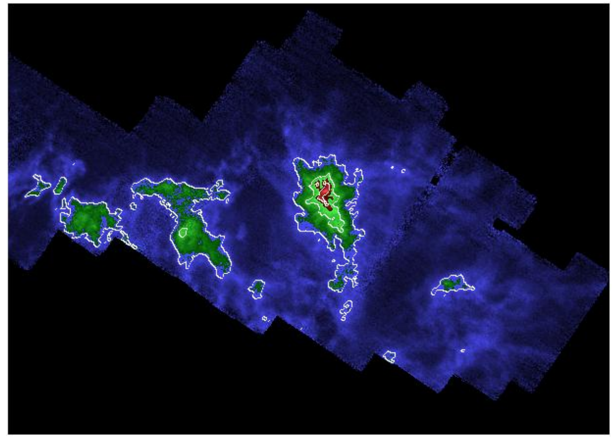


(Sánchez, Alfaro & Pérez 2005)

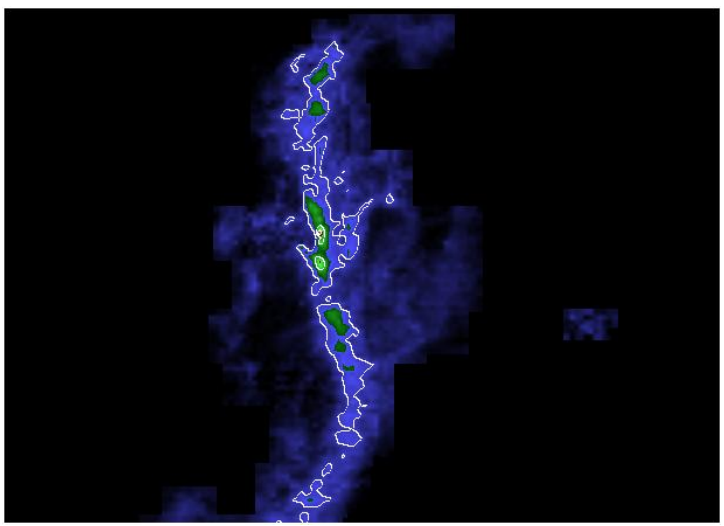
Application to Galactic clouds

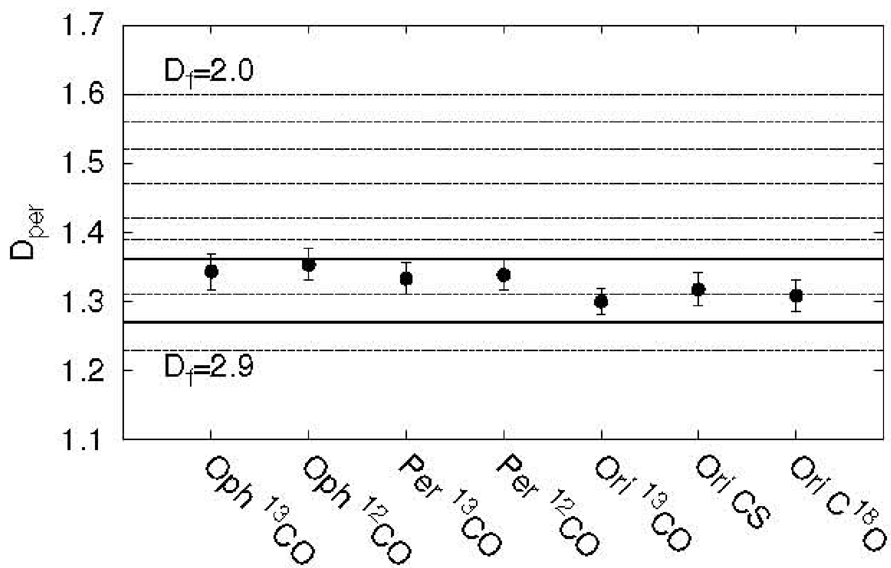




13CO maps

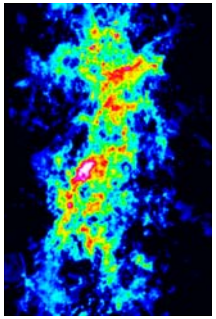
Ophiuchus, Perseus (COMPLETE, Ridge et al. 2006) Df=2.6-2.8 (Universal?)





Sánchez, Alfaro & Pérez 2007

Orion (Nobeyama, Tatematsu et al. 1993)

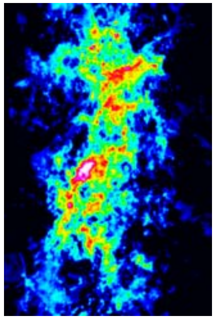


Do all MW clouds have the same D_f?

- $D_{per,}$ for molecular clouds in the Milky Way, ranges between 1.2 and 1.6. This transforms to 2.2 -2.9 for D_f (Bazell & Desert 1988; Dickman et al. 1990; Scalo 1990; Falgarone et al. 1991; Hetem & Lepine 1993; Vogelaar & Wakker 1994; Stutzki et al. 1998; Lee 2004, Lee et al. 2008)

- The average value is 1.35 , but with large dispersion (Elmegreen & Scalo 2004)

- For Galactic clouds at different molecular transitions, but analyzed in a homogeneous way, this range shortens to $D_{per,}$ (1.3-1.4) -> Df (2.6-2.8) (Sánchez et al. 2005, 2007)



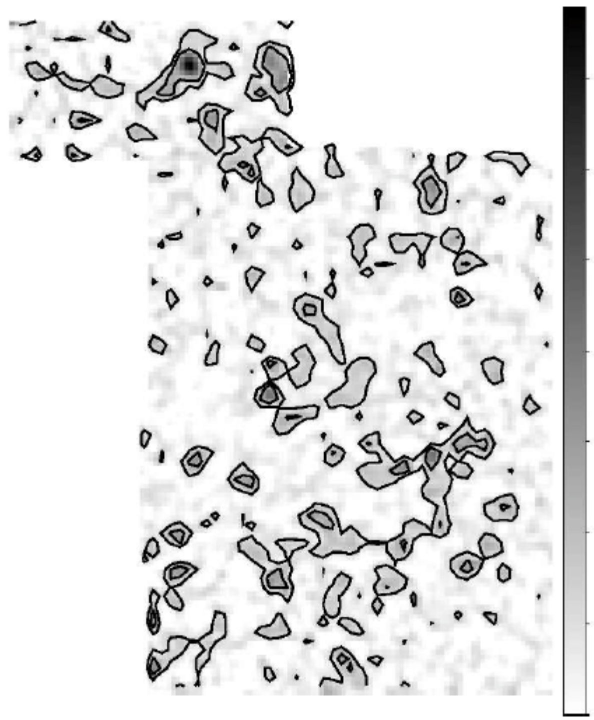
CO in M33

Molecular clouds in other galaxies appear to show different values of D_{per} (LMC, M33, etc.) .

Molecular Clouds

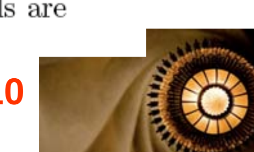
D_{per}=1.65±0.06
D_f ~1.6-1.8 << 2.6

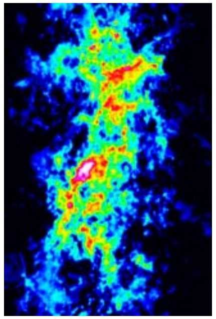
(Sánchez et al. 2010a)



(Rosolowsky et al. 2007)

Fig. 2.— Integrated intensity of CO emission in the central region of M33 (Fig. 1). The gray scale runs linearly from 0 (white) to the maximum value (black, ∼ 7.8 K km s^{-1}). Contour levels are shown at 1 and 2 K Km s^{-1}.

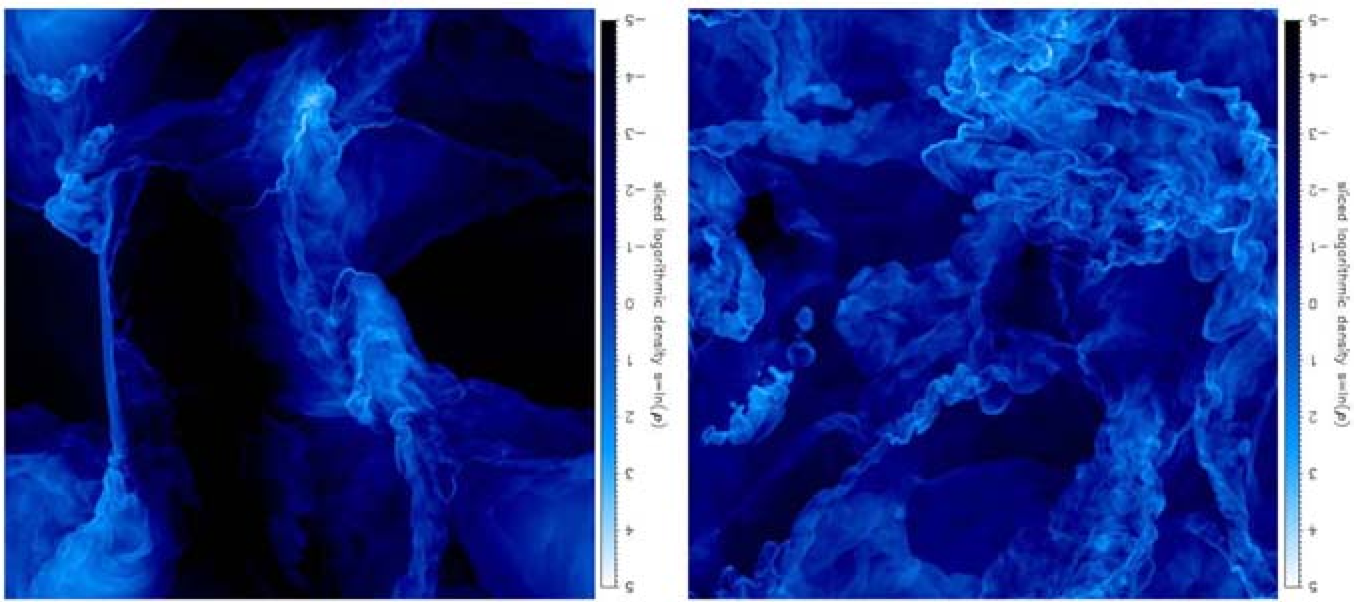




Fractal Dimension & Turbulence

Stochastic forcing term **f** as source term in equations of hydrodynamics: **compressive modes (x f = 0), and solenoidal (nabla · f = 0)** (Federrath et al. 2009, 2010)

D▪2.3 for compressive and D▪2.6 for solenoidal forcing (M=5.5)



Stellar Clusters

a) Is the spatial distribution of the recent born stellar clusters similar to that of the molecular clouds where they were formed?

b) How does the spatial distribution of embedded cluster compare with the distribution of pre-stellar cores? Do they show a different mass function?

c) What fraction of young clusters are destroyed when loosing the embedding parental gas? Does it depend on cluster internal properties, on local environment or/and on average properties of the host galaxies?

d) How do the survivors evolve with time on the phase space?

Two approaches: Galactic Cluster Systems and Resolved Clusters

Cluster Systems in External Galaxies

- Similar approach to Dynamics of Populations in life sciences

- Mass, age (photometry & spectroscopy) and position (astrometry) of clusters in external galaxies (i.e. Goddard et al. 2010; Elmegreen & Hunter 2010; Portegies Zwart 2010; Adamo et al. 2010; Whitmore et al. 2010).

- Provide information about the mechanisms eroding the clusters and about the fraction of cluster surviving infant mortality. But it needs some important assumptions about the initial mass cluster function, and the history of the cluster formation rate (Konstantopoulos et al. 2010). Observational data can be biased by several factors. Results are sensitive to data handling (Goddard et al. 2010).

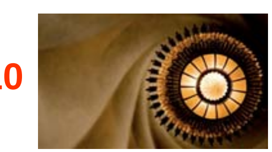
- Two main family of disruption models:
    - Mass Dependent Disruption (MDD) (Boutloukos & Lamers 2003)
    - Mass Independent Disruption (MID) (Fall, Chandar & Whitmore 2005)

Cluster Systems in External Galaxies: Some Recent Results

1. Strong correlation between the percentage of stars within bound clusters (Γ) and the star formation rate density (Σ) (Goddard et al. 2010). Good agreement with Proszkow & Adams (2009) simulation.

$$\Gamma(\%) = (29.0 \pm 6.0)\Sigma_{SFR}^{0.24\pm0.04}(M_{\odot}\ yr^{-1}\ kpc^{-2})$$

2. Cluster disruption by tidal interaction with the stellar-gas complex where the cluster has been formed; cluster local environment controls its destiny (Elmegreen & Hunter 2010). A new approach to the disruption models.



Clusters Resolved into Stars (I)

Data:

- Position on the Sky
- Kinematic Data (Radial Velocity and Proper Motions) (LAMOST, GAIA, etc)
- Multi-wavelength photometry (SDSS)

Physical Properties:

- Cluster Membership
- Luminosity Function and hence Mass Function
- Distance and Age
- Stellar Distribution on the Phase Subspaces
- If Kinematic Data are good enough $\longrightarrow$ Dynamical Mass

Clusters Resolved into Stars (II)

Cluster Signatures to be Compared with models

- Mass Segregation
- Clumpiness Strength or Radial Spatial Profile
- Clumpy or Filamentary Structures versus Smoothed Kinematic Distributions

Tools

- Cluster Membership estimation (parametric & non-parametric methods) [Cabrera-Caño & Alfaro 1985, 1990; Sánchez et al. 2010a]
- Q Structure Parameter based on MST (Cartwright & Whitworth 2004)
- Correlation Fractal Dimension (unbiased) [Sánchez & Alfaro 2007, 2008, 2009]
- Mass segregation, MST based parameter (Allison et al. 2010)

Evolution of Internal Structure of Galactic Clusters: Sample & Method

- Cluster selection mainly based on quality and extent of proper motion data.

- Sample (N=16; Age [7Myr-3Gyr]; d < 4 kpc)

- Membership analysis based on non-parametric methods.

- Determination of the MST (Q) parameter for the cluster members.

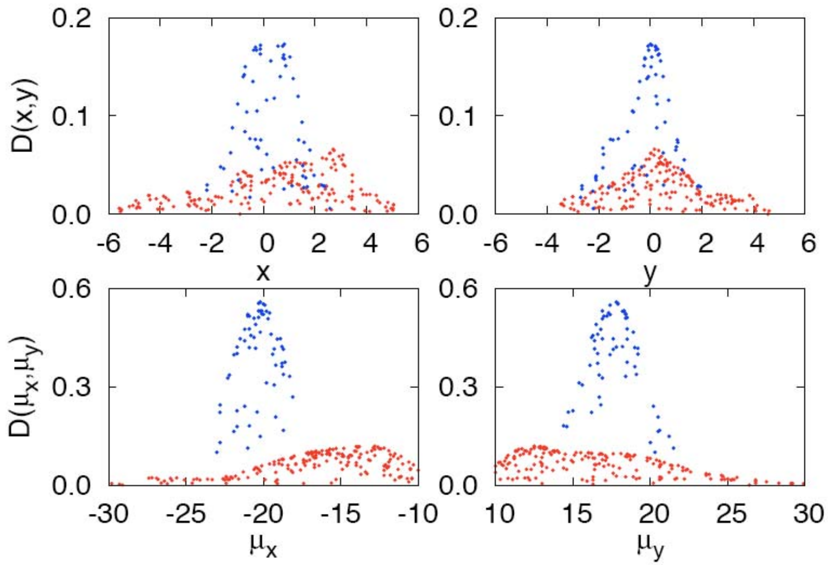
$$Q = \frac{\bar{m}}{\bar{s}}$$

Mean length of the branches (normalized)

Mean separation between stars

Evolution of Internal Structure of Galactic Clusters: Cluster Membership

IC 2391 (fractal distr.)



NGC 2194 (radial distr.)

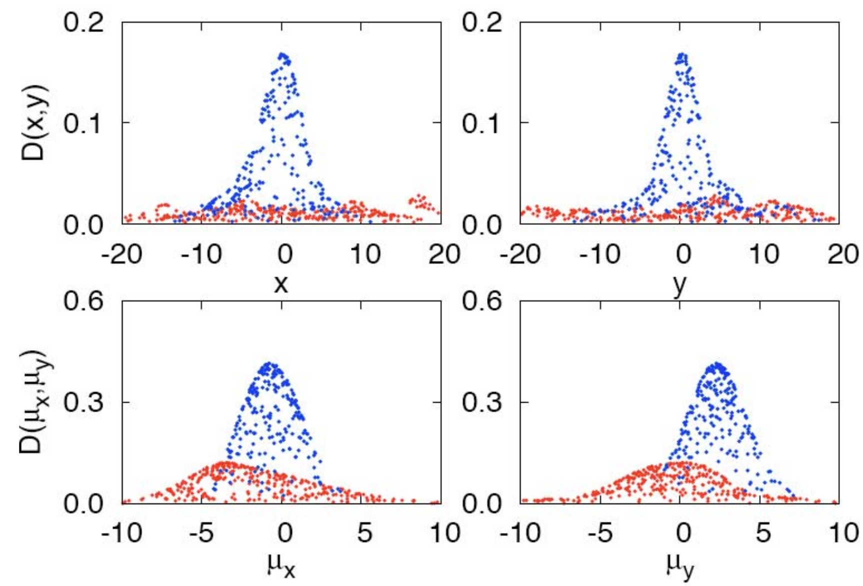
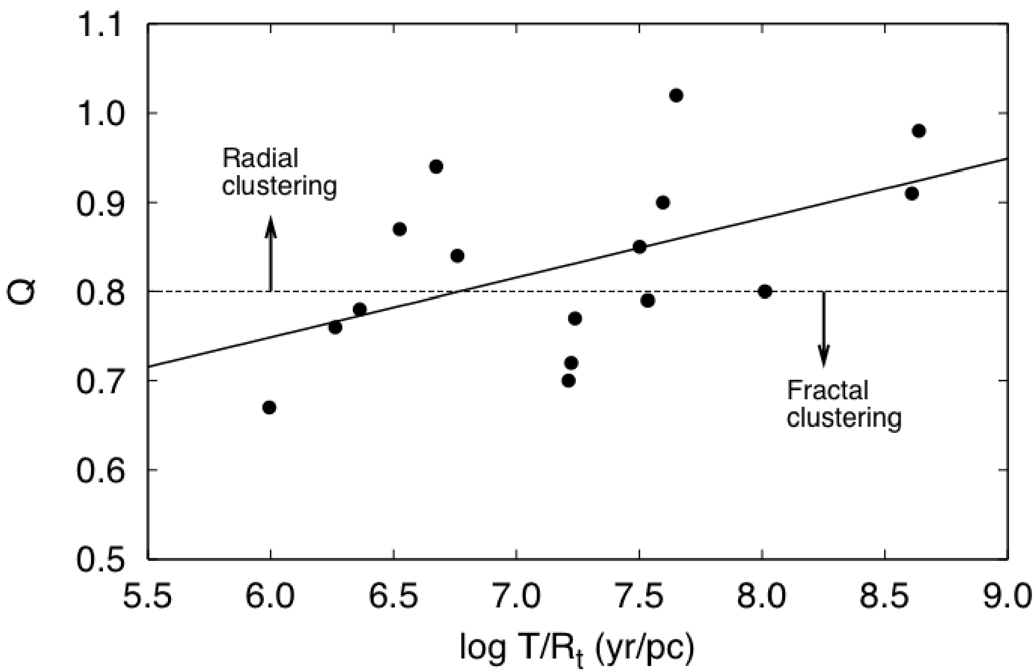


Fig. 3.— Probability density funcions for the stars in the region of the open cluster IC 2391. The two upper panels show the projections in x and y of the probability densities in the position space. The two lower panels are the projections in μ_x and μ_y of the probability densities in the proper motion space. Red circles refer to field stars and the blue ones to cluster members.

.— Same as Fig. 3, but for the open cluster NGC 2194.

Evolution of Internal Structure of Galactic Clusters: Results

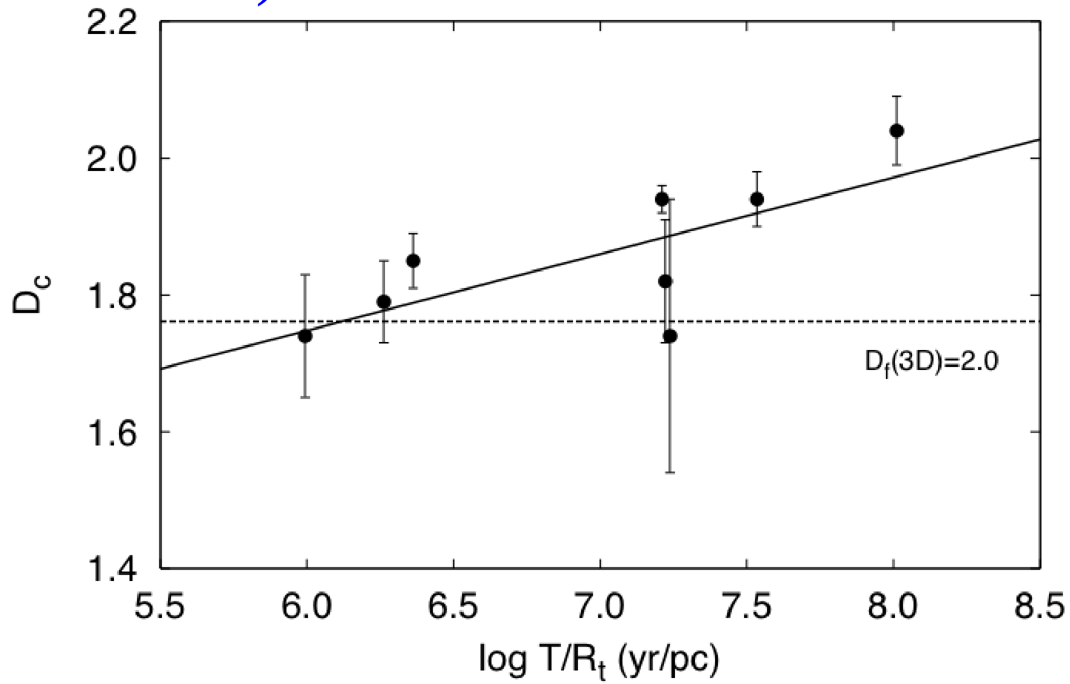
Weak correlation between Q and age



Evolution of Internal Structure of Galactic Clusters (Results)

Strong correlation between D_c and age for clusters with Q < 0.8 (clumpy structure)

(Sánchez & Alfaro 2009)



Evolution of Internal Structure of Galactic Clusters (Results)

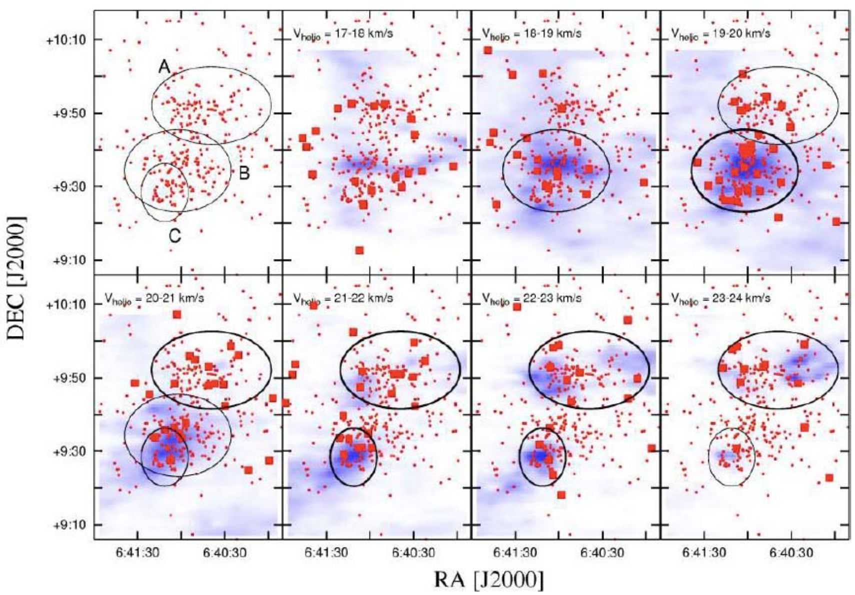
- Galactic clusters tend to erase clumpy structure with age.

- Clumpy strength of fractal clusters decreases over time, but clusters as old as a few hundred million years can still (or again) show fractal patterns.

- Young fractal clusters can show D_c values below the one estimated for molecular clouds in the solar vicinity.

Kinematic Studies

• NGC 2264 and Orion Nebula are the only stellar clusters, in the optical range, where some kind of structure has been observed in the radial velocity distribution (Furesz et al. 2006, 2008).

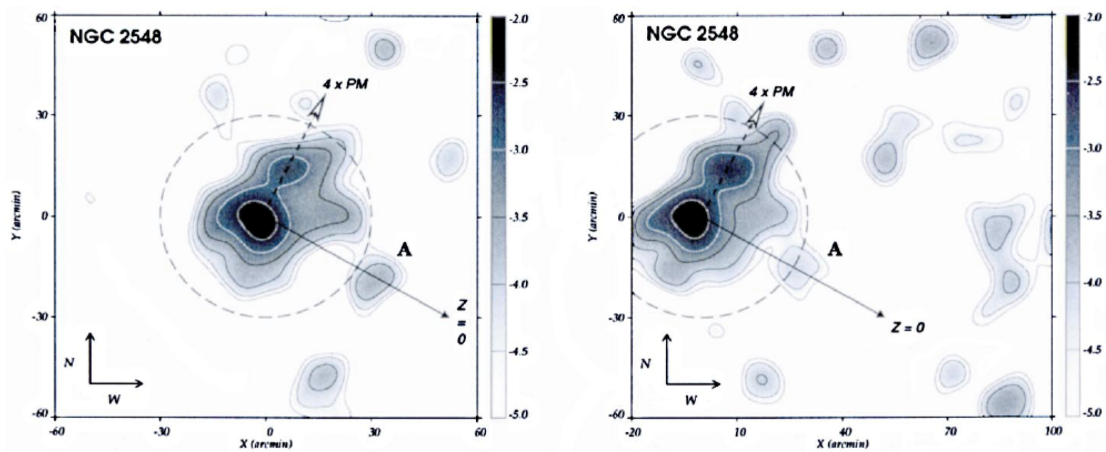


Stellar groupings and molecular cores appear to be coincident in space and velocity.

Age ~ 1.5 – 3 Myr

Kinematic Studies

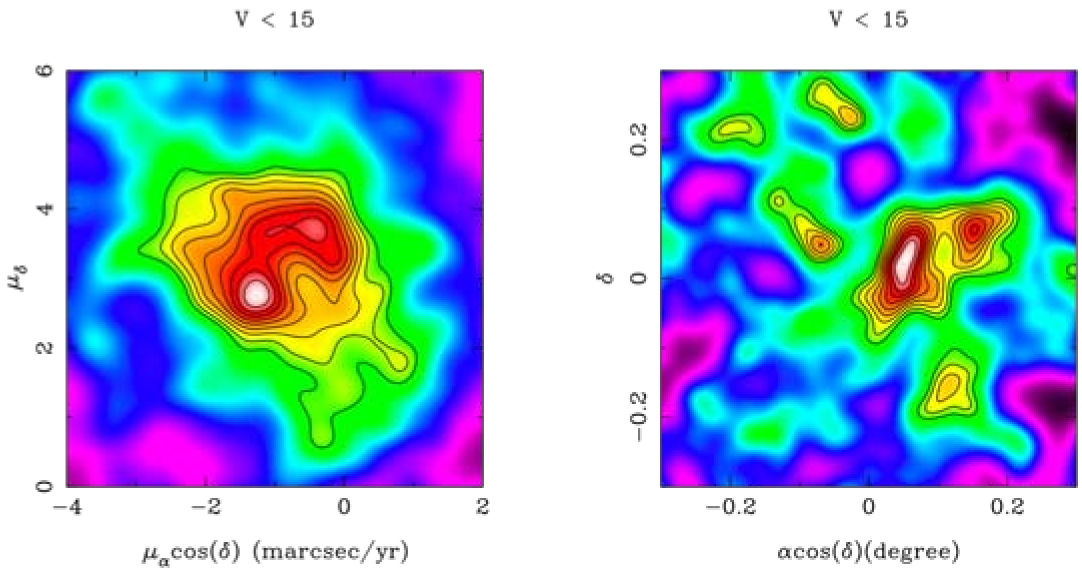
- NGC 2148 shows a clumpy structure with three well defined blobs (C1, C2, A) according to Bergond et al. (2001).

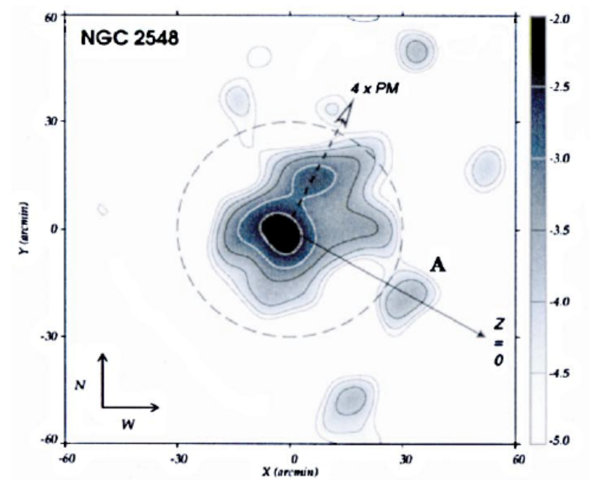


Cluster included in a new astrometric catalog based on "La Carte du Ciel" (Royal Navy Observatory, Spain) (Vicente et al. 2010a)

Kinematic Studies (NGC 2148)

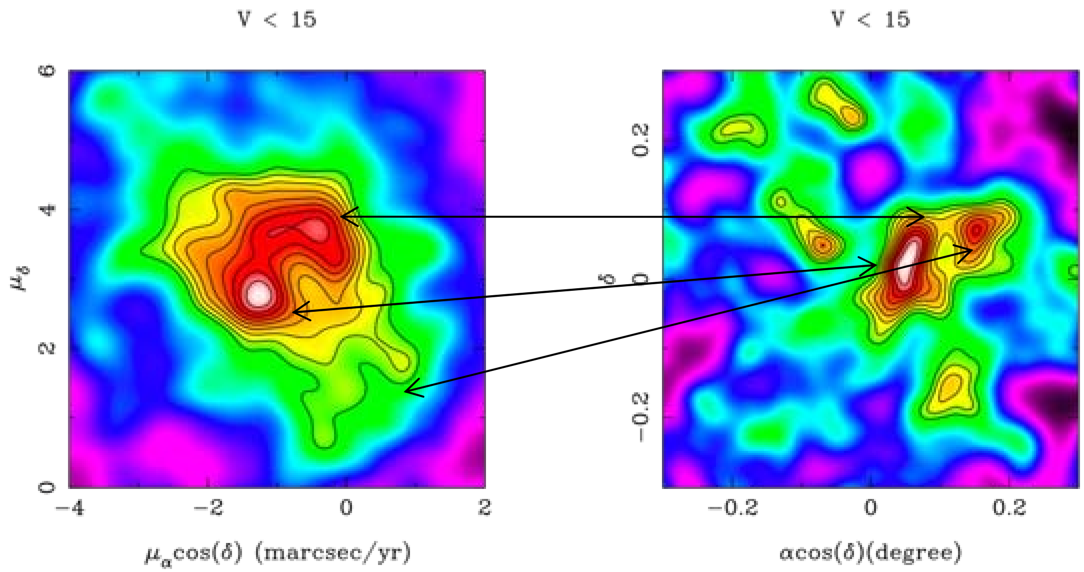
- Based on the new astrometric catalog we have analyzed the internal structure of the cluster in the phase space.





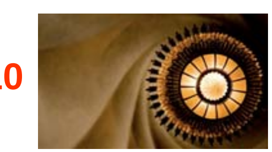
(Vicente et al. 2010b, in preparation)

Kinematic Studies (NGC 2148)



- The clumpy structure observed on the spatial map translates into the proper motion distribution
- Supervirial velocity
- Feature A appears to be not connected to NGC 2148
- Mass segregation

(Vicente et al. 2010b; in preparation)



Future Work

Our group has started a program to analyze the structure of stellar clusters on the phase space, whose main guidelines are:

- Selection of clusters with high probability of being in supervirial regime. This will allow us to work with currently available proper motion catalogs.

- Prepare statistical tools for analyzing future large astrometric catalogs.

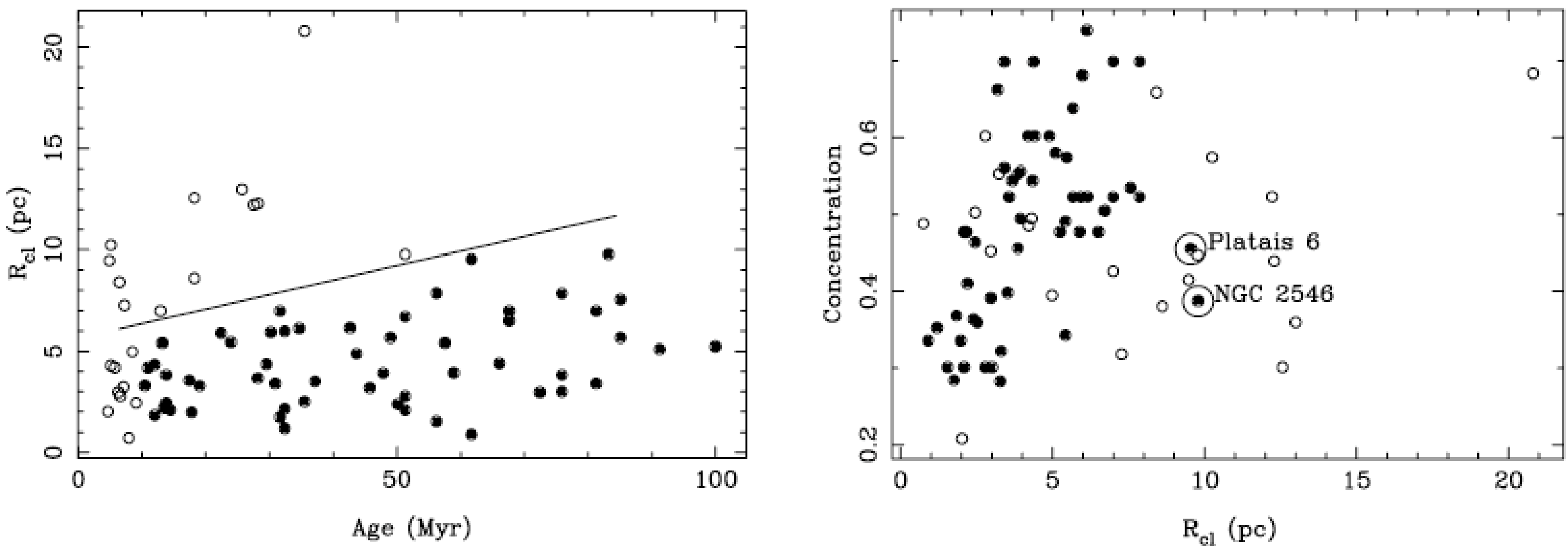
Thanks to:

Néstor Sánchez (in charge of these objectives in the group); Belén Vicente & Enrique Pérez (collaborators)

Carlos Román-Zuñiga, Antxon Alberdi & Miguel Pérez-Torres for discussions

SOC & LOC of the meeting for invitation

Cluster Selection



(Elias et al. 2009; Alfaro et al. 2010)