Galactic Warp and Flares

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Introduction

In these notes I have put together some ideas about the project of modeling the warp and flare of our Galaxy. First I try a simple transformation that involves displacing points along the direction orthogonal to the disk. Although this is easily accomplished, it is shown that this warping transformation introduces shape distrotions. In particular, circular orbits are distorted into ellipses. Since this is not dynamically justified, a second approach to warping the disk is tried. The second approach is just a tilt applied to rings. It is shown that this second warping transformation does not distort shapes. The explicit transformation functions for both, positions and velocities are found and their use is illustrated by warping a Miyamoto-Naga model.

The Warp

First we are going to create some mathematical mappings that allows us to "warp" either potential or density functions, or particle positions, to generate a warp. This will be accomplishe by just shifting points vertically according to a warping function z_{warp} that gives the vertical shift as a function of the R coordinate on the disk. The dependence is that of a power law (R^{α}), and such that $z_{warp}(R_2) = z_2$.

We define the following parameters:

- R_1 is the 2D radius where the warp will begin.
- R_2 is the 2D radius where the warp will finish.
- z_2 is the height that the warp has achieved at R_2.
- α is a power index that controls the shape of the warp.

Here is the warp function:

$$z_{warp}(R;\ R_1,\ R_2,\ z_2,\ \alpha) = \begin{cases} 0, & \text{for } R \leq R_1 \\ z_2\ ((R - R_1)\ /\ (R_2 - R_1))^{\alpha}, & \text{for } R > R_1 \end{cases} \qquad (1)$$

This is a function that bends the z = 0 axis upwards using a power-law.

Here is a plot of the warp function for $R_1 = 1$, $R_2 = 2$, $z_2 = 1/2$ and α equal to 1, 2, 3 and 4 (top to bottom curves on the positive R side).

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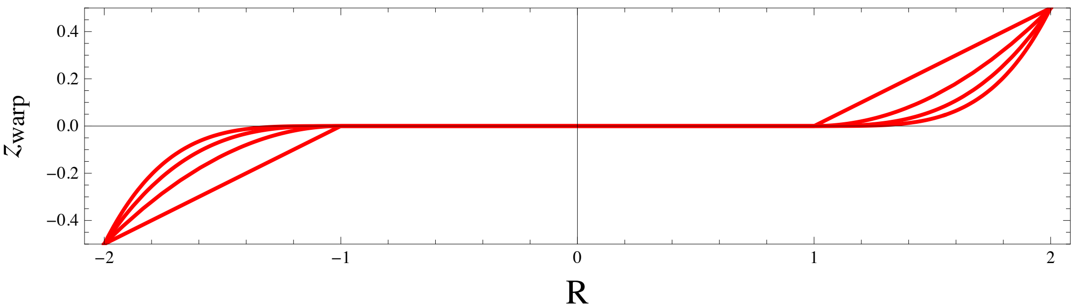

Figure 1. Warping function. See text for details.

Notice that for α=1 (linear case), the R-derivative of the function is discontinuous at the place where the warp begins. All other values produce continuous slopes.

The inclination angle can be easily computed as just as the ArcTan of the slope, which is the derivative of this function:

$$\text{inc}\,(R;\ R_1,\ R_2,\ z_2,\ \alpha) = \text{ArcTan}\left(\frac{dz_{warp}}{dR}\right) = \text{ArcTan}\left(\frac{z_2}{R_2 - R_1} \times \left(\frac{R - R_1}{R_2 - R_1}\right)^{\alpha-1}\right) \quad (2)$$

We can now introduce the azimuthal dependence to produce a continuous warp of a disk. For this we can use a simple *cosine* function whose argument is the galactocentric azimuthal coordinate φ displaced from the origin by an amount φ_o which corresponds to the direction of the maximum height of the warp. If we had chosen a *sine* function, then it would be the direction of the line of nodes:

$$z_{fullwarp}\,(R,\ \varphi) = z_{warp}\,(R;\ R_1,\ R_2,\ z_2,\ \alpha) \times \cos\,(\varphi - \varphi_o) \quad (3)$$

Here is a plot for the case: $R_1 = 1/4$, $R_2 = 2$, $z_2 = 1$, α=3.

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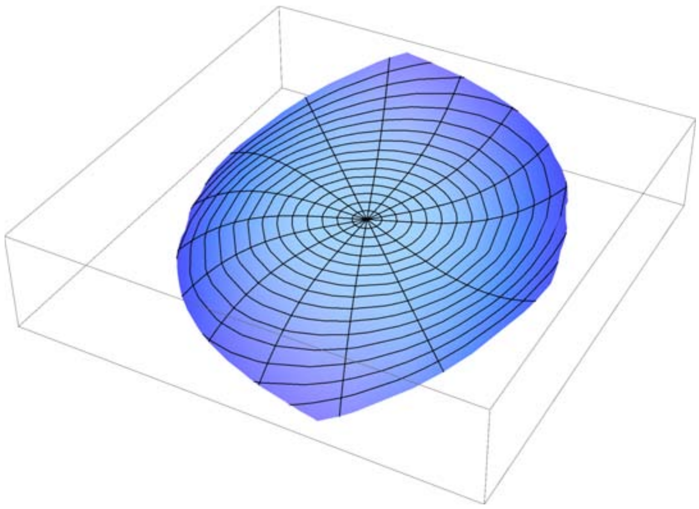

Figure 2. Example of a warped disk.

This warping function can be applied directly as an additive factor for the z-ccordinate of the density or potential functions of an axisymmetric model. Here is an example with the Miyamoto-Nagai model:

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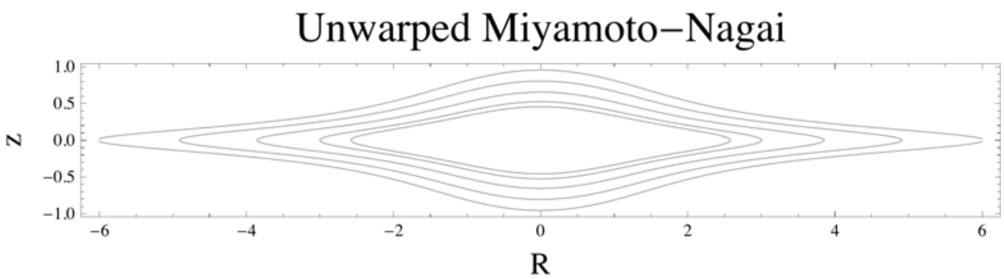

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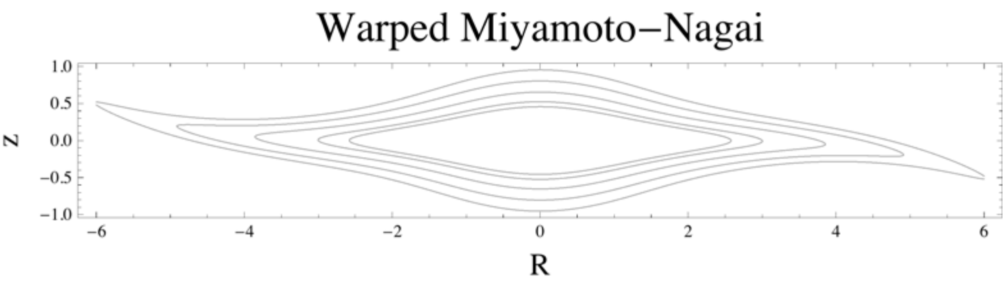

Figure 3. Comparison of warped and unwarped Miyamoto-Nagai model.

It can also be applied to the z-ccordinate of an ensemble of particles. However, the velocity vectors should also be rotated and this is a bit more complicated. But instead of doing this, we are going to try a second approach because of the problem discussed in the next subsection.

■ Problems with this approach

The approach we have presented in this section has a serious drawback: it produces spatial distortions. The reason is that it displaces points along the vertical direction only, and this distorts shapes. For instance, a circle on the unwarped galactic plane is distorted into an ellipse whose axis ratio increases with the warp. See the following figure, where a ring has been transformed into an elongated elliptical ring by using a purely vertical displacement, like the one used in our approach.

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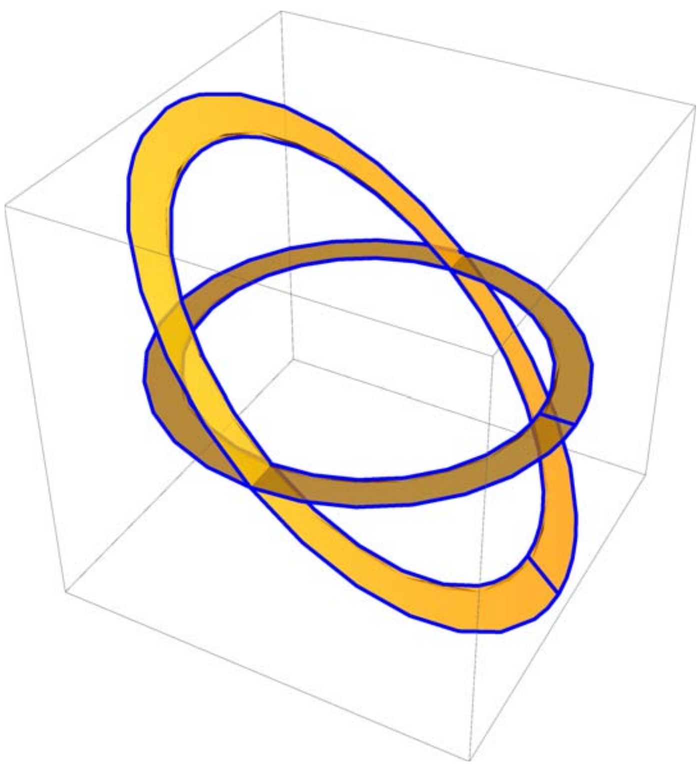

Figure 4. Distortion introduced by vertical shifting. The horizontal circular ring is elongated by this transformation in the direction orthogonal to the line of nodes. This transformation does not preserve spatial shapes.

The Warp: 2nd approach.

It is clear that the way to achieve a proper warp that preserves shapes is by tilting on annuli, like in the following figure.

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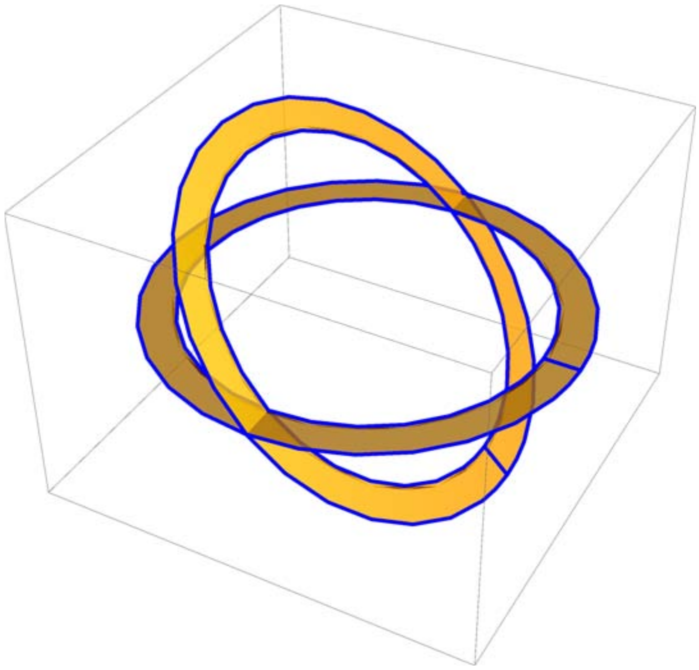

Figure 5. Tilt transformation. In this case no distortion is introduced.

A tilt, like the one shown here, does not introduce shape distortions, so this is the approach that we will use.

▪ Transformation equations : spatial part

We now derive the equations needed to achieve this tilt transformation.
Let's begin with a single ring at fixed radius R and fixed tilt angle ψ. The geomtery is shown in the next figure.

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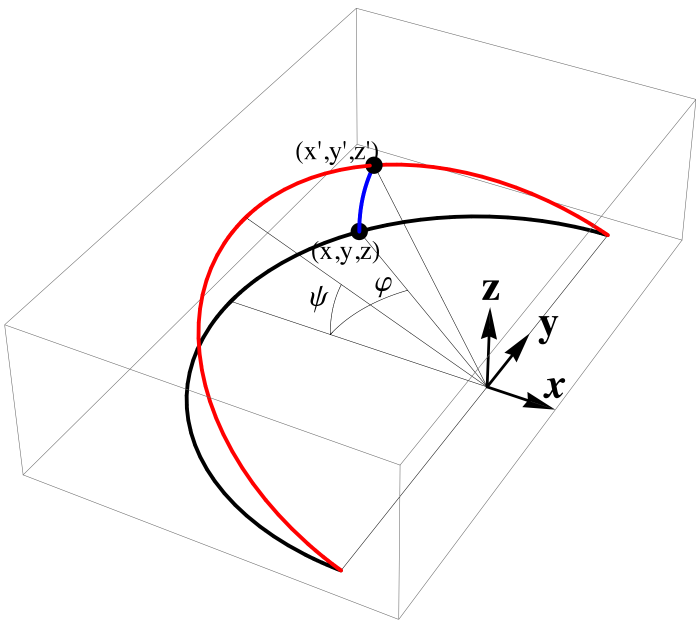

Figure 6. Geometry of the tilt transformation. It is a right-hand rotation along the positive y-axis by an angle ψ. The angle φ is the azimuth along the ring. The original coordinates are (x, y, z), the transformed coordinates are: (x', y', z').

The cartesian coordinates of a circle or radius R on the $z = 0$ plane are just:

$$x = R\cos(\varphi), \quad y = R\sin(\varphi), \quad z = 0, \tag{4}$$

where φ is just the azimuthal angle which runs from 0 to 2π. The positive x-axis corresponds to $\varphi = 0$.

To acomplish a rotation of an angle ψ using the y-axis as pivot and in the sense that this is a clockwise rotation when seen from the positive y-axis (see figure 6), we multiply by the following rotation matrix:

$$\mathbf{A} = \begin{pmatrix} \cos(\psi) & 0 & \sin(\psi) \\ 0 & 1 & 0 \\ -\sin(\psi) & 0 & \cos(\psi) \end{pmatrix} \tag{5}$$

The tilted cartesian coordinates are then:

$$\mathbf{r' = Ar} = \begin{pmatrix} \cos(\psi) & 0 & \sin(\psi) \\ 0 & 1 & 0 \\ -\sin(\psi) & 0 & \cos(\psi) \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x\cos(\psi) + z\sin(\psi) \\ y \\ -x\sin(\psi) + z\cos(\psi) \end{pmatrix} \tag{6}$$

Applying this transformation to the ring (equations 4), we get the coordinates of the tilted ring:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{ring} = \begin{pmatrix} R\cos(\varphi)\cos(\psi) \\ R\sin(\varphi) \\ -R\cos(\varphi)\sin(\psi) \end{pmatrix} \tag{7}$$

So, we now only need to specify the tilt angle as a function of the cylindrical R coordinate. For this we use a function like that used in the first approach (equation 1), except that we are now dealing with angles, rather than displacements.

$$\psi(R;\ R_1,\ R_2,\ \psi_2,\ \alpha) = \begin{cases} 0, & \text{for } R \le R_1 \\ \psi_2\left((R - R_1)/(R_2 - R_1)\right)^{\alpha}, & \text{for } R > R_1 \end{cases} \tag{8}$$

Remember that the tilt is applied beyond R_1. The resulting warp is such that the tilt angle increases as a power law whose exponent is α and such that at R_2 it has a value equal to ψ_2.

Here is an example for a warped galaxy with $R_1 = 1, R_2 = 2, \psi_2 = \pi/8$, α=3. The edge of the disk is at R_{max}=2. We show below this galaxy using rings.

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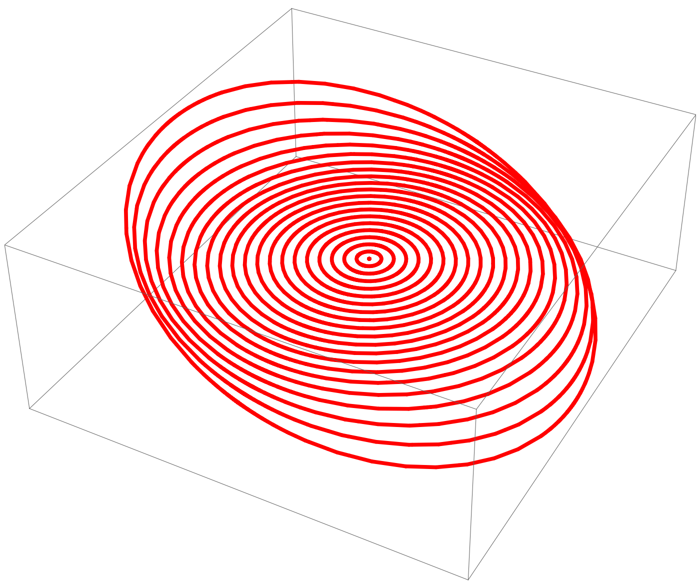

Figure 7. Circular orbits on a warped disk accomplished using the warping function given by equation (8). The parameters of the warp are: $R_1 = 1, R_2 = 2, \psi_2 = \pi/8$, α=3.

The rings correspond to circular orbits on the warped disk. We can clearly see here that, unless there is differential precession of the rings, the warp will be maintained and won't evolve into a flaring. For a flaring to develop we need to have an external torque on the rings, such as that exerted by a non-spherical halo.

Here is the same example as before, but now represented as a surface.

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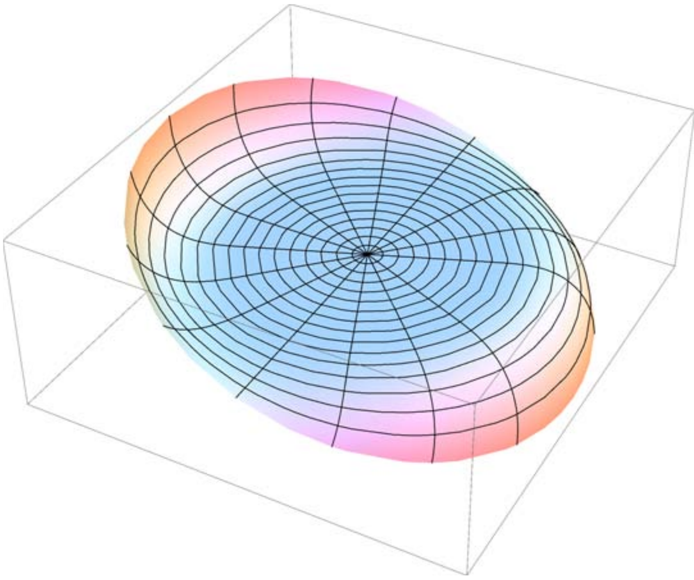

Figure 8. Same as figure (7), but now the disk is represented as a 2-D surface.

■ Transformation equations : kinematical part

Having accomplished a proper warping that produces no spatial distortion of the orbits, we must now find the way to tilt the corresponding kinematics.

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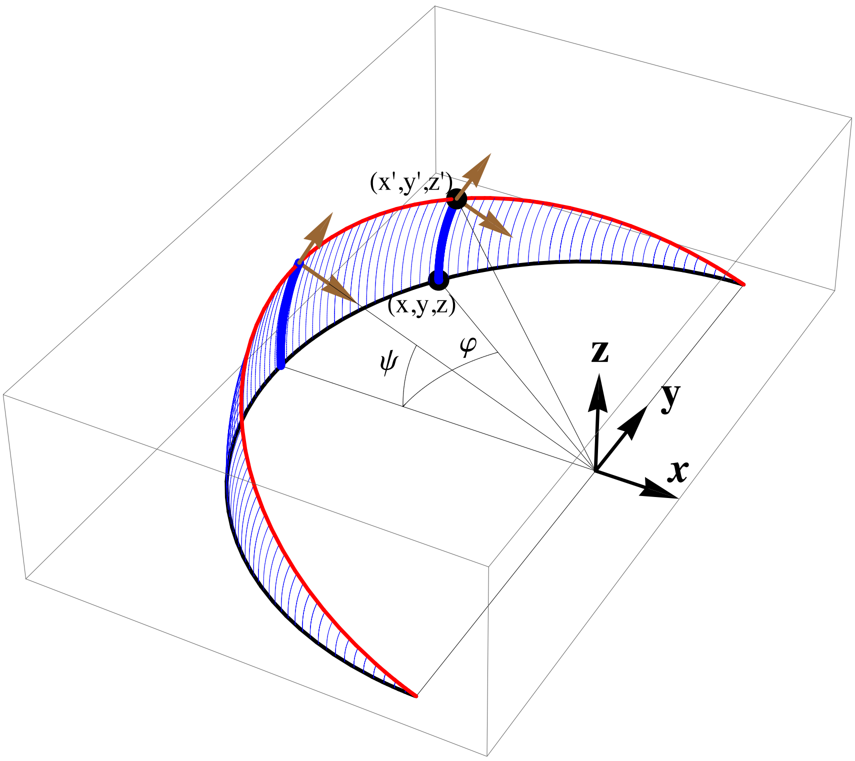

Figure 9. Geometry of the tilt transformation applied to the kinematics. Notice that the tilt produces a rotation of the velocity vectors in the x-z plane. The y component of the velocities is not affected.

Looking at the figure we see that the y-component of the velocity is not affected by the tilt transformation (remember, the y-axis is the rotation axis), while the x and z velocity components are rotated by an angle ψ in the clockwise direction, when seen from the positive y-axis.

If (v_x, v_y, v_z) are the original velocities in the unwarped model and (v'_x, v'_y, v'_z) are the transformed velocities in the warped model, the transformation equations are given by:

$$\begin{pmatrix} v_x' \\ v_y' \\ v_z' \end{pmatrix} = \begin{pmatrix} v_x \cos(\psi) + v_z \sin(\psi) \\ v_y \\ -v_x \sin(\psi) + v_z \cos(\psi) \end{pmatrix} \tag{9}$$

which we recognize as the same transformation applied to positions (equation 6).

The transformation has been applied in cartesian coordinates, because the tilt, being a simple rotation along a cartesian axis, results in a simple mathematical transformation. If we had used cylindrical coordinates, then the transformation equations from cylindrical to cartesian coordinates should first be used. In the case of velocities, they are not trivial (see figure 9).

Application of the tilt-warp transformation.

The tilt-warp tharnsformation can be applied to a continuous 3D function, like a density model or a potential function, or to a set of phase-space coordinates of an ensemble of particles.

In the first case the transformation is:

$$f(x(x', y', z'), y(x', y', z'), z(x', y', z')) = f(x', y', z') \tag{10}$$

where we need the inverse transform of equation (6):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{r} = \mathbf{A}^{-1} \mathbf{r}' = \begin{pmatrix} \cos(\psi) & 0 & -\sin(\psi) \\ 0 & 1 & 0 \\ \sin(\psi) & 0 & \cos(\psi) \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} x' \cos(\psi) - z' \sin(\psi) \\ y \\ x' \sin(\psi) + z' \cos(\psi) \end{pmatrix} \tag{11}$$

and we have used the fact that, **A**, being a matrix that reperesnets a rotation, is orthogonal and thus its inverse is equal to its transpose.

In the case of phase-space coordinates, the transformation is simply:

$$\left(x'(x, y, z),\ y'(x, y, z),\ z'(x, y, z);\ v_x'(v_x, v_y, v_z),\ v_y'(v_x, v_y, v_z),\ v_z'(v_x, v_y, v_z)\right) \tag{12}$$

where the transformation equations are given by equations (6) and (9).

Here is an example in which we warp the Miyamoto-Nagai model of figure (3).

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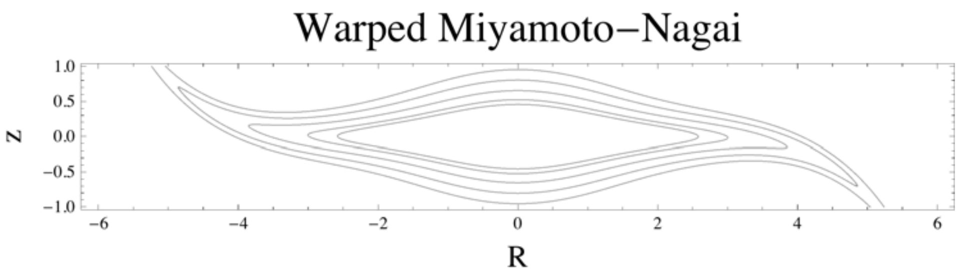

Figure 10. Warped Miyamoto-Nagai model using the tilt-warp transformation. We have used $R_1 = 2$, a tilt angle of $\psi_2 = \pi/8$ at $R_2 = 6$, and an exponent $\alpha = 3$.