Modelling the Galaxy

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Why models?

- Selection effects
  - The stars we see are determined by our location
  - Many are obscured by dust, which is has an extremely complex distribution
- Errors
  - Their locations in phase space are uncertain
  - Measured parallaxes can be –ve
  - Parallaxes determine 3/6 phase-space coordinates
  - Hopeless to consider that “Gaia will return all 6 phase-space coordinates”
- Models enable us to allow for selection effects and errors
- Models enable us to synthesize information from different surveys

Traditional models

- Assume f(x,v) = $\sum_\alpha \rho_\alpha$(x) exp[-v^2/2$\sigma_\alpha{}^2$(x)]
- Adopt a dust model
- Exploit that for SSP can predict distribution in L, colours, etc for stars of given (τ,Z) parameterised by $\mathcal{M}$
- Then produce mock catalogues from isochrones and limiting magnitude etc of catalogue
  - Besancon model (Creze+ 1986 – Robin+ 2003) imposes a degree of self-consistency between τ, σ_z and z_0
  - TRILEGAL model (Groenewegen+ 2002, Girardi+ 2005)
  - Galaxia (Sharma+ 2010) a more efficient machine for sampling a pdf of stars
  - The Gaia mock catalogue now has this foundation

My themes

- We need to upgrade from ρ(x) exp(-$v^2/2\sigma^2$) to fully dynamical f(x,v)
- In addition to generating mock catalogues we need to fit DF to data by generating pdf in space of observables
  - (l,b,ϖ,μ_α,μ_δ,v_{los},V, V-I,T_{eff},logg,..)

Why dynamical models?

- A primary target is DM distribution.
  - We infer this by using $d^2x/dt^2 = -\nabla\Phi$
  - We want to know how the Galaxy works:
  - E.g. the relation between distribution of stars in velocity space and spiral structure/the bar
  - Cause of the warp
- We want our models to be dynamically consistent – the same tomorrow as today

Why equilibrium models?

- Can get DM density only to the extent that the Galaxy is in equilibrium
- At r<20 kpc expect system to be close to equilibrium
- Assumption of dynamical equilibrium dramatically reduces the freedom of models – it makes the model-fitting problem satisfyingly over-determined
- We can model non-equilibrium phenomena by perturbing an equilibrium model
- Also should use equilibrium models to identify signature in data of non-equilibrium structures (streams, warp, spirals, etc)

Why models based on actions?

- Jeans theorem encapsulates loss of freedom on restriction to equilibrium: $f(x,v) \rightarrow f(I_1, I_2, I_3)$
- Actions are uniquely favoured integrals:
  - Adiabatic invariants
    - Useful during slow changes in Φ
    - Enable us to identify orbits in neighbouring potentials
  - Easy to understand physically
    - range $(0,\infty)$
  - Can be used as momenta of a canonical coordinate system
    - conjugate variables the angles θ
  - (θ,J) the natural coordinates of perturbation theory
  - $d^3x\, d^3v=(2\pi)^3\, d^3J$ so $f(J)$ density of stars in 3d action space

Why models with known DF?

- Coordinates of individual stars of no significance
  - it is the density of stars that carries information
- Since stars are distinguishable by age, mass, [Fe/H], [α/Fe],…. we need many DFs f(J)
- On account of selection effects and errors we must fit model in space of observables
  - (l,b,ϖ, μ_α, μ_δ, v_{los}, V, V-I, T_{eff}, logg, [Fe/H],…)
  - 11d and counting
- n graduations per axis $\rightarrow$ n^d cells, >30 stars/cell $\rightarrow$ 1.5e9 stars in catalogue
- Even with Gaia barely feasible because getting optimal grid will be extremely hard
- But suppose have N-body model
  - Great majority of particles will be too far from Sun enter mock catalogue, so need >>10^9 particles to get Gaia-sized catalogue
- People usually project catalogue to low-d subspace and grid that
  - Projection erases correlations between variables but correlations are key

- If we have f_α(J) with α labelling population, can predict pdf(l,b,ϖ,μ_α, μ_δ, v_{los}, V,V-I,T_{eff}, logg, [Fe/H],…) and calculate L(data)

Oxford models

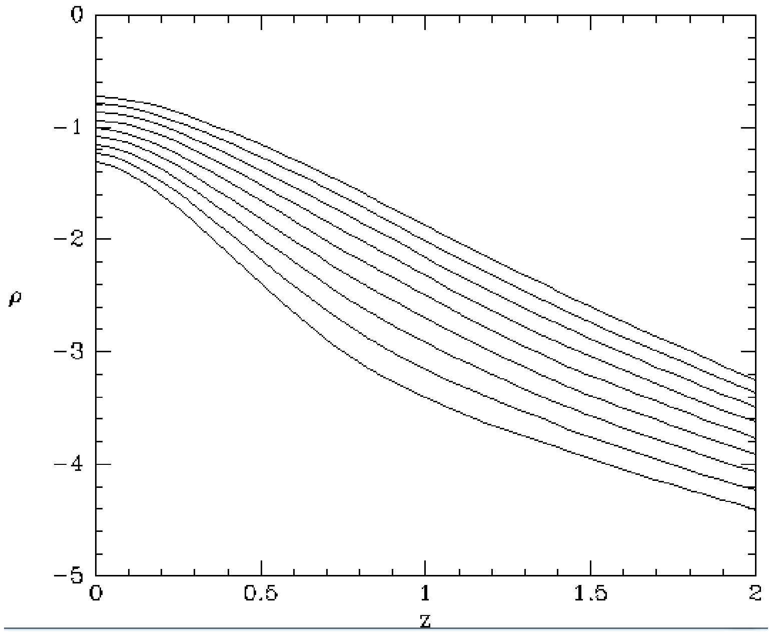
- Construct f(J) of discs from "quasi-isothermal"
- Assume f(J) and so far no use of isochrones; emphasis on computing likelihood of data (McMillan)
- f(J) assembled from "quasi-isothermal" building blocks

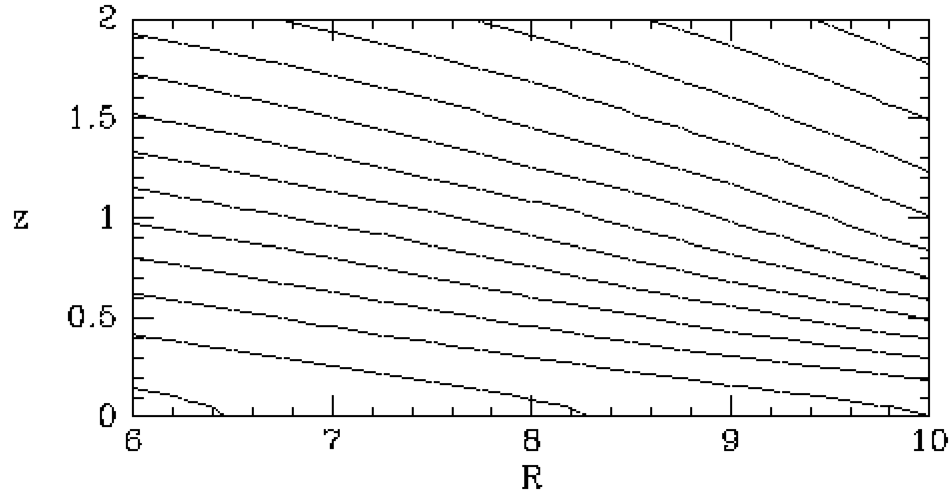
$$f_{\rm thk}(J_r, J_z, L_z) = f_{\sigma_r}(J_r, L_z) f_{\sigma_z}(J_z)$$

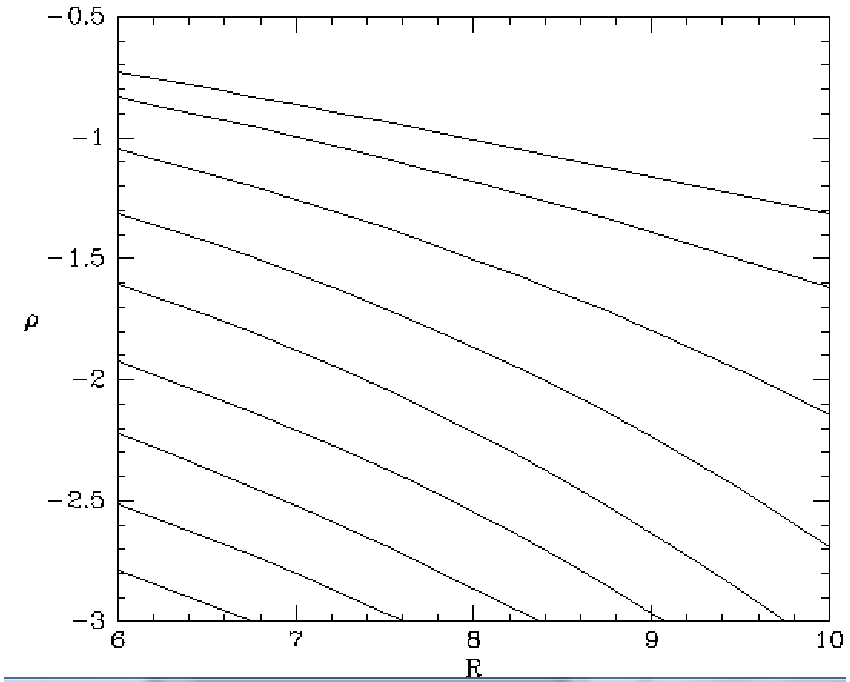
$$f_{\sigma_r}(J_r, L_z) \equiv \left. \frac{\Omega\Sigma}{\pi\sigma_r^2\kappa} \right|_{R_c} [1 + \tanh(L_z/L_0)] \mathrm{e}^{-\kappa J_r/\sigma_r^2}$$

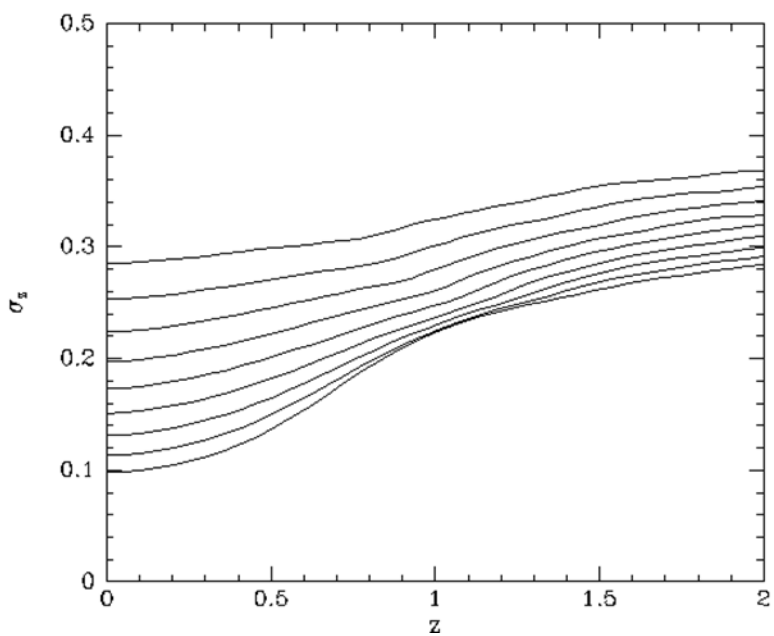
$$f_{\sigma_z}(J_z) \equiv \frac{\nu}{2\pi\sigma_z^2} \mathrm{e}^{-\nu J_z/\sigma_z^2}$$

A quasi-isothermal model

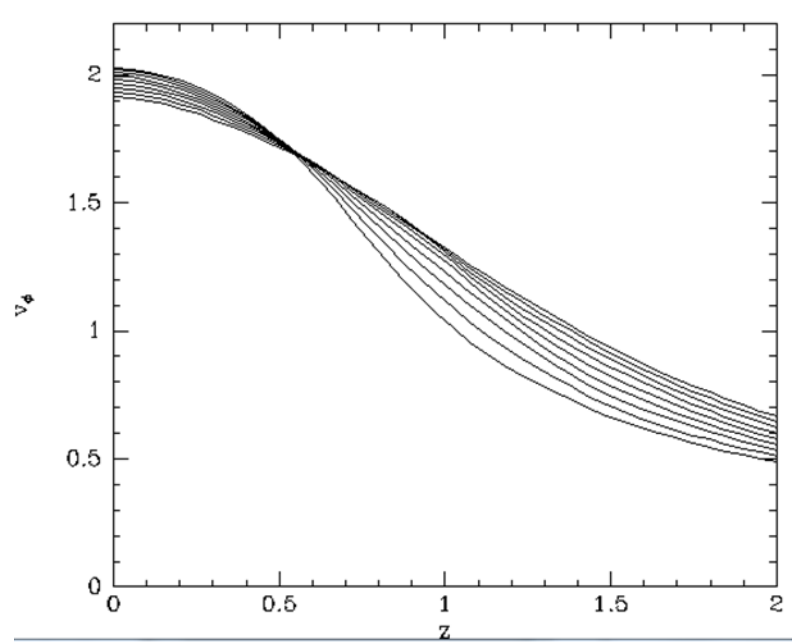








0.5
0.4
0.3
0.2
0.1
0
σ_z
0
0.5
1
1.5
2
z



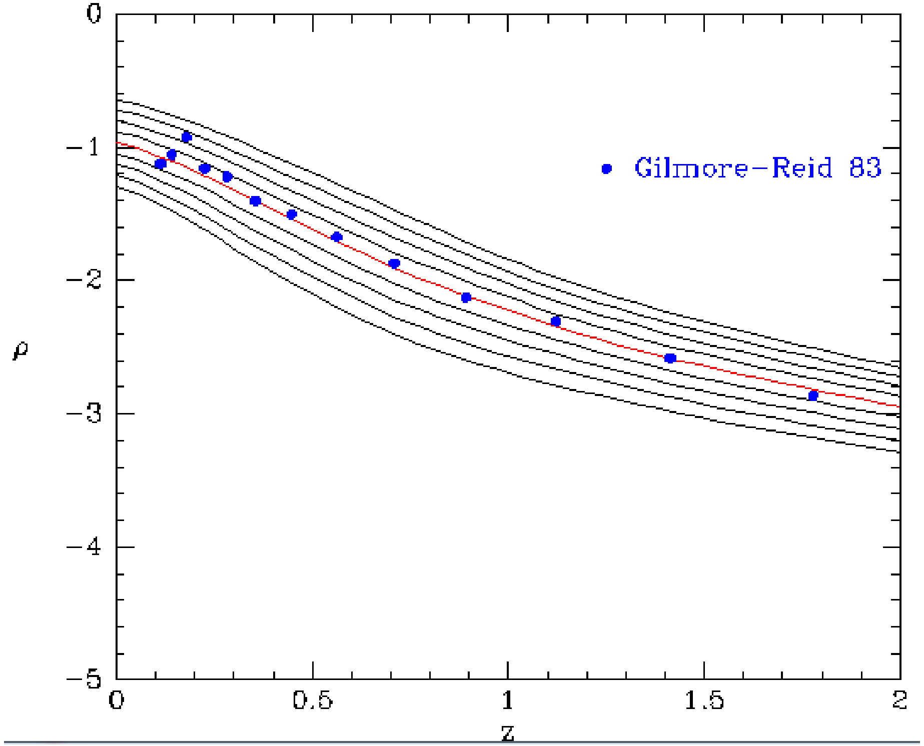
2
1.5
1
0.5
0
v_ϕ
0
0.5
1
1.5
2
z

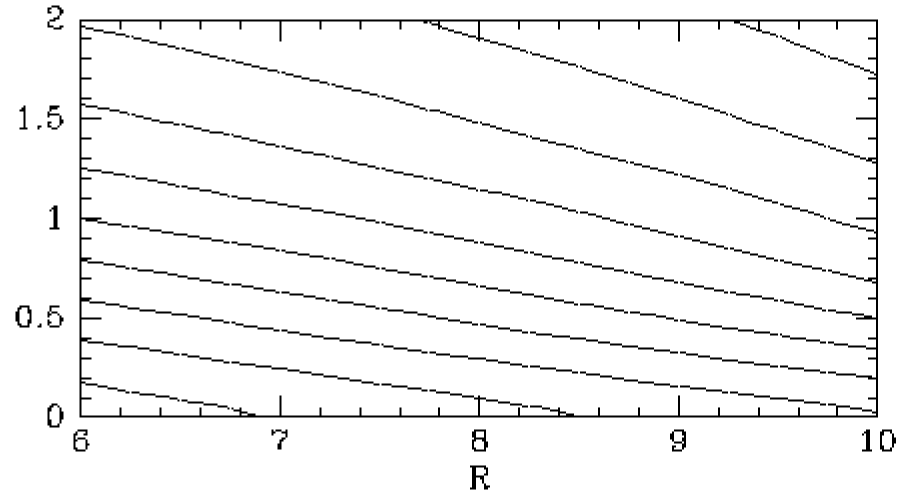
A model with a growing thin disc

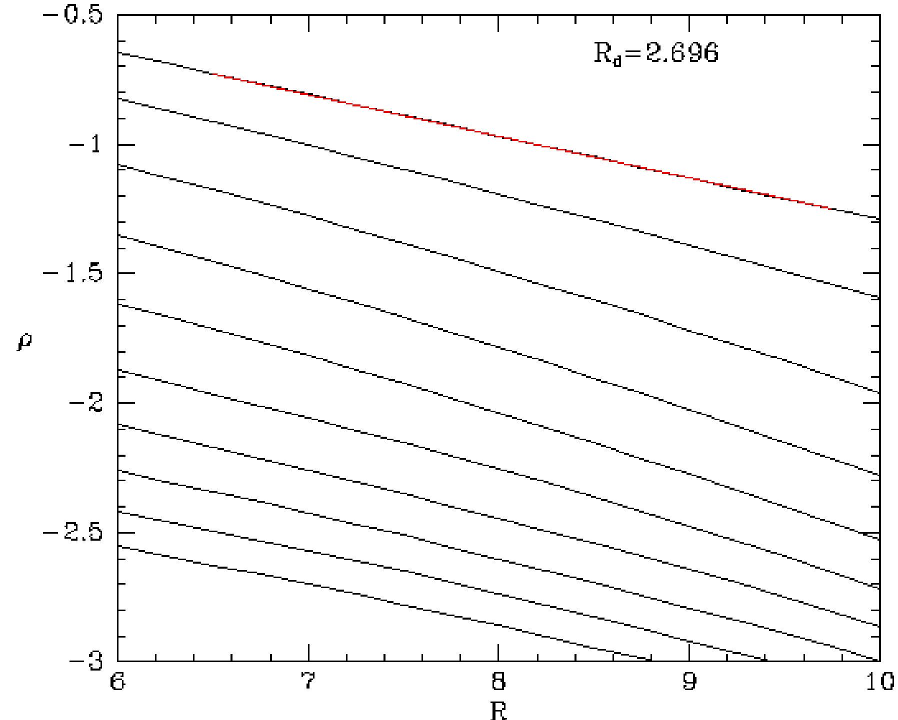
$$\sigma_z(L_z,\tau) = \sigma_{z0}\left(\frac{\tau+\tau_1}{\tau_{\rm m}+\tau_1}\right)^{\beta} {\rm e}^{q(R_0-R_{\rm c})/R_{\rm d}}$$

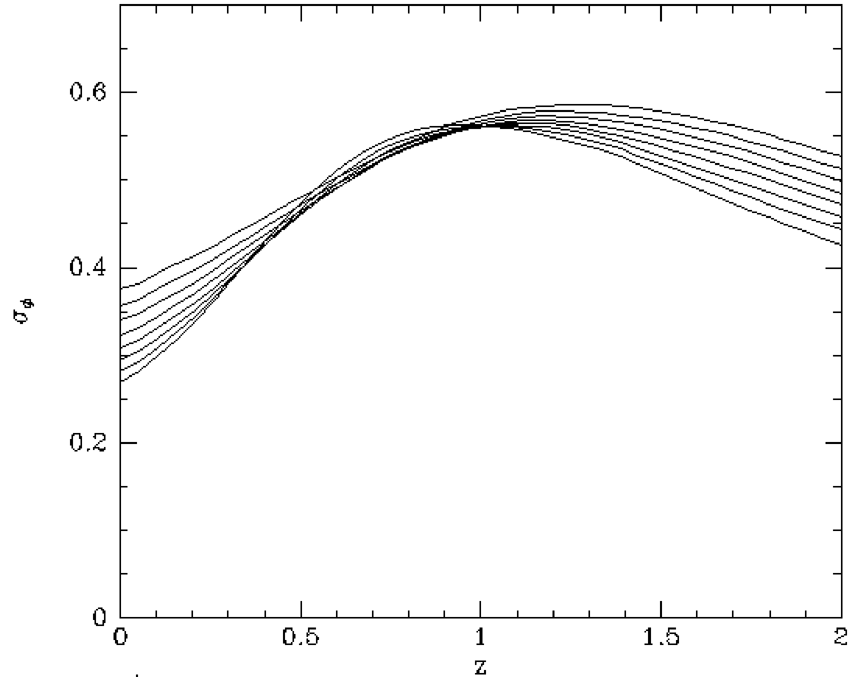
$$f_{\rm thn}(\mathbf{J}) = \int_0^{\tau_m} {\rm d}\tau\, \frac{{\rm e}^{\tau/t_0} f_{\sigma_r}(J_r,L_z) f_{\sigma_z}(J_z)}{t_0({\rm e}^{\tau_{\rm m}/t_0}-1)}$$

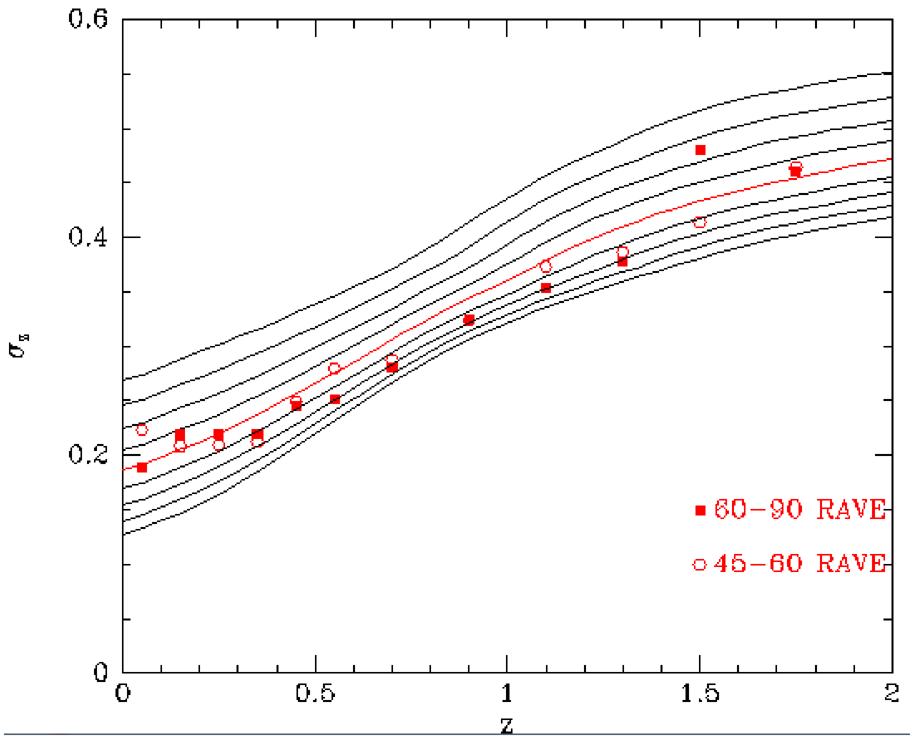
A “realistic” model

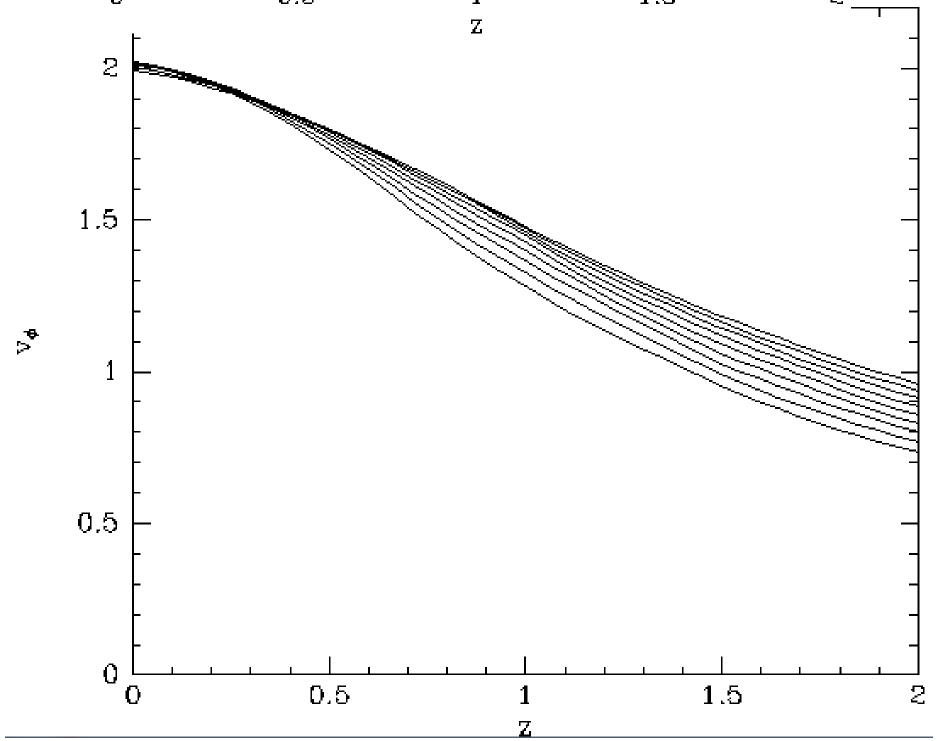


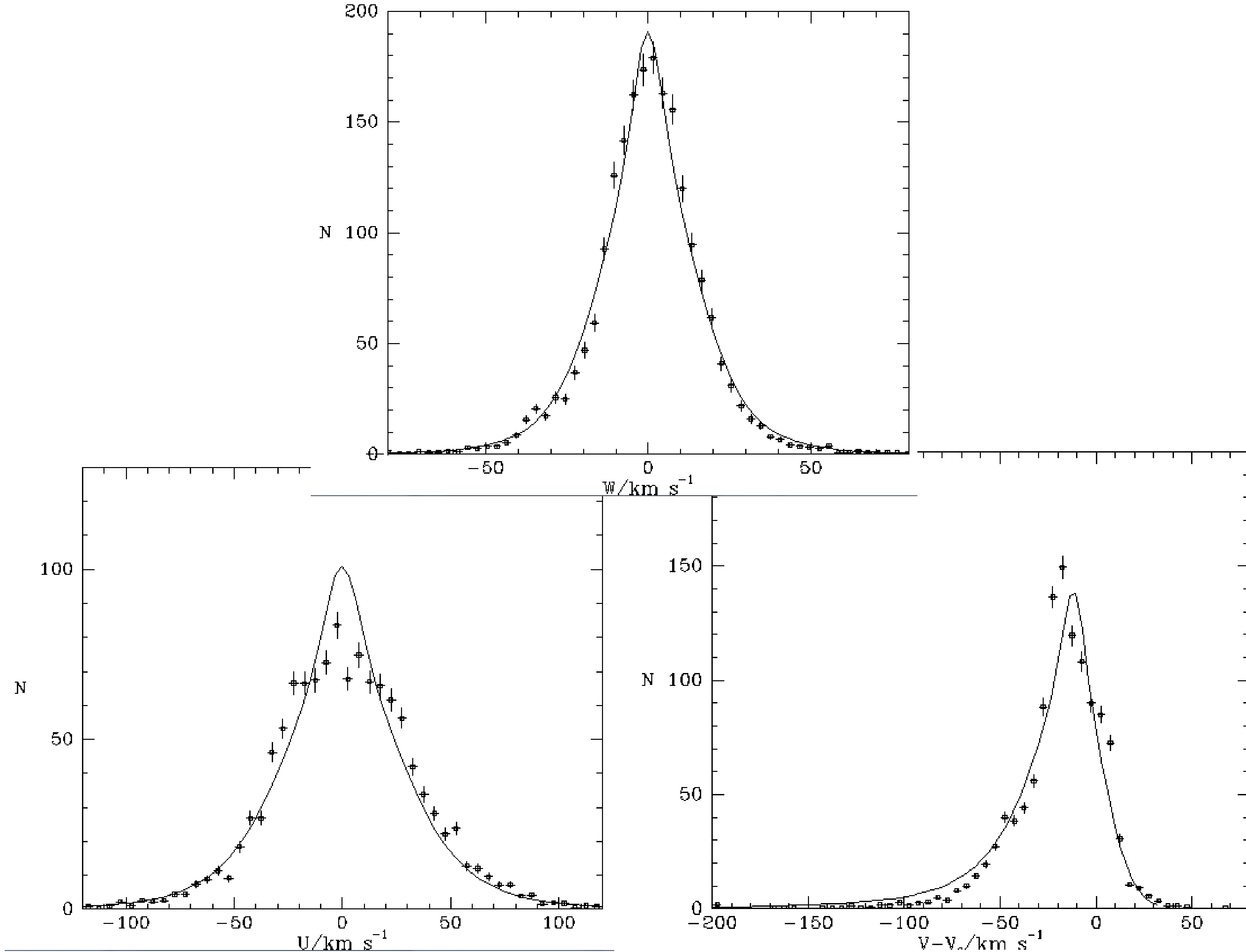





σφ
z
0
0.2
0.4
0.6
0.5
1
1.5
2


σz
z
60−90 RAVE
45−60 RAVE
0
0.2
0.4
0.6
0.5
1
1.5
2


vφ
z
0
0.5
1
1.5
2



200
150
N
100
50
0
−50
0
50
W/km s⁻¹
100
N
50
0
−100
−50
0
50
100
U/km s⁻¹
150
N
100
50
0
−200
−150
−100
−50
0
50
V−V⊙/km s⁻¹

Next steps

- Add chemical dimension by making parameters functions of Z in addition to τ
- Things to do
  - Fit DF to data in space of observables (McMillan) & discover how precisely we can define physics with data of various qualities
  - Fit DF to N-body models to be sure it can represent reasonable chemodynamical histories
  - Use model as intermediary between N-bodies & data (Brown, Velasquez & Aguilar 2005)?
  - Use model as intermediary between N-bodies and Mock Gaia?

Conclusions

- We should upgrade front end of Mock-Gaia to chemo-dynamical models
- In addition to Mock Catalogues we need the underlying pdf in space of observables
- Think of Mock-Gaia as a filter between dynamics and space of observables
- The dust model in this filter should be refined by Gaia data
- We don't yet know what physics questions can be answered with given data
- I think we know how to find out though
- But we don't have so much time
- Fitting models based on good physics will be a powerful means of detecting systematics
- So let's have continuous updating of the Gaia products!