Measuring the MW's gravity **without** dynamical models

Jorge Peñarrubia (IAA-CSIC)

in collaboration with Sergey Koposov & Matt Walker

Barcelona 2012

Measuring the MW's gravity **without** dynamical models

Barcelona (unspecified time in the future)

Question

Dynamical methods:

1) solve the eqs. of motions in a model potential, under a give Law of Gravity

2) fit observed motion of stellar bodies

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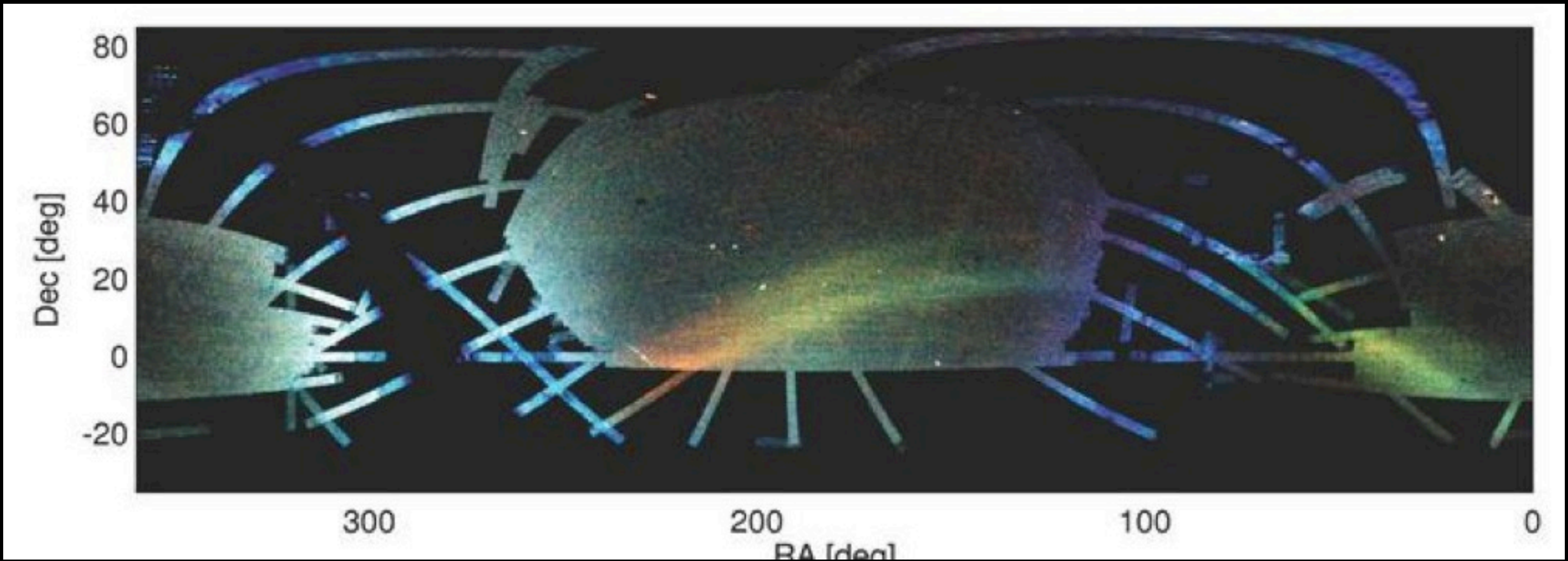
Our goal:

measure potential and test Gravity simultaneously *without* solving the eqs. of motion

Tidal streams!



Belokurov + 06

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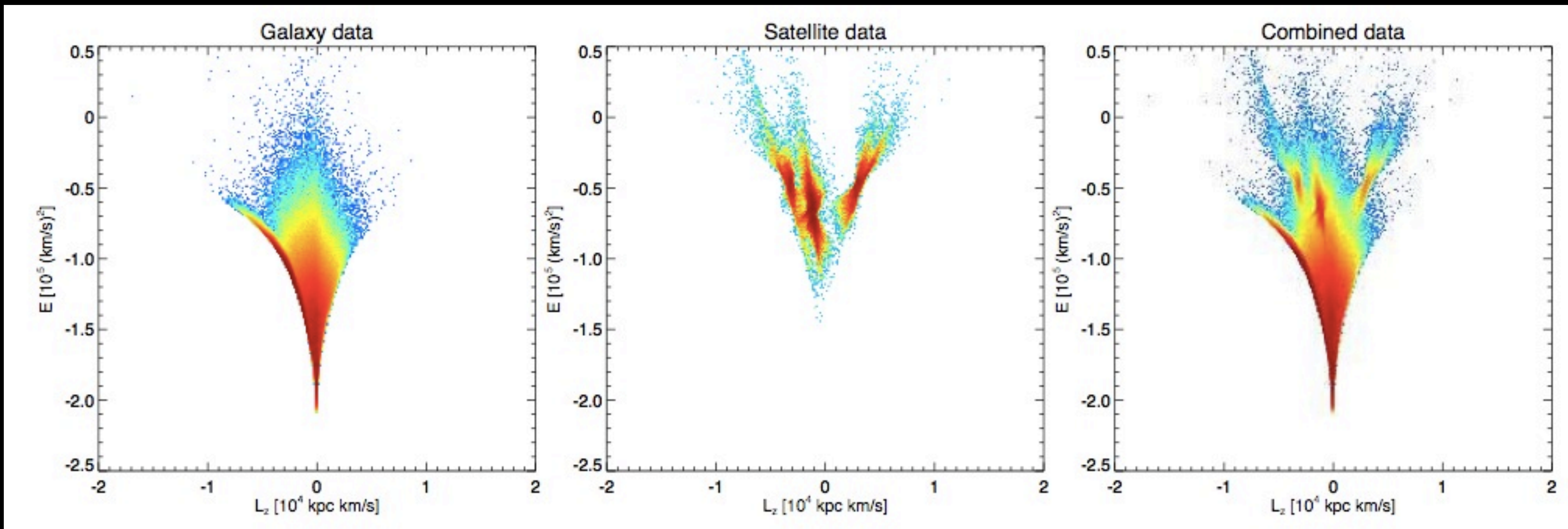


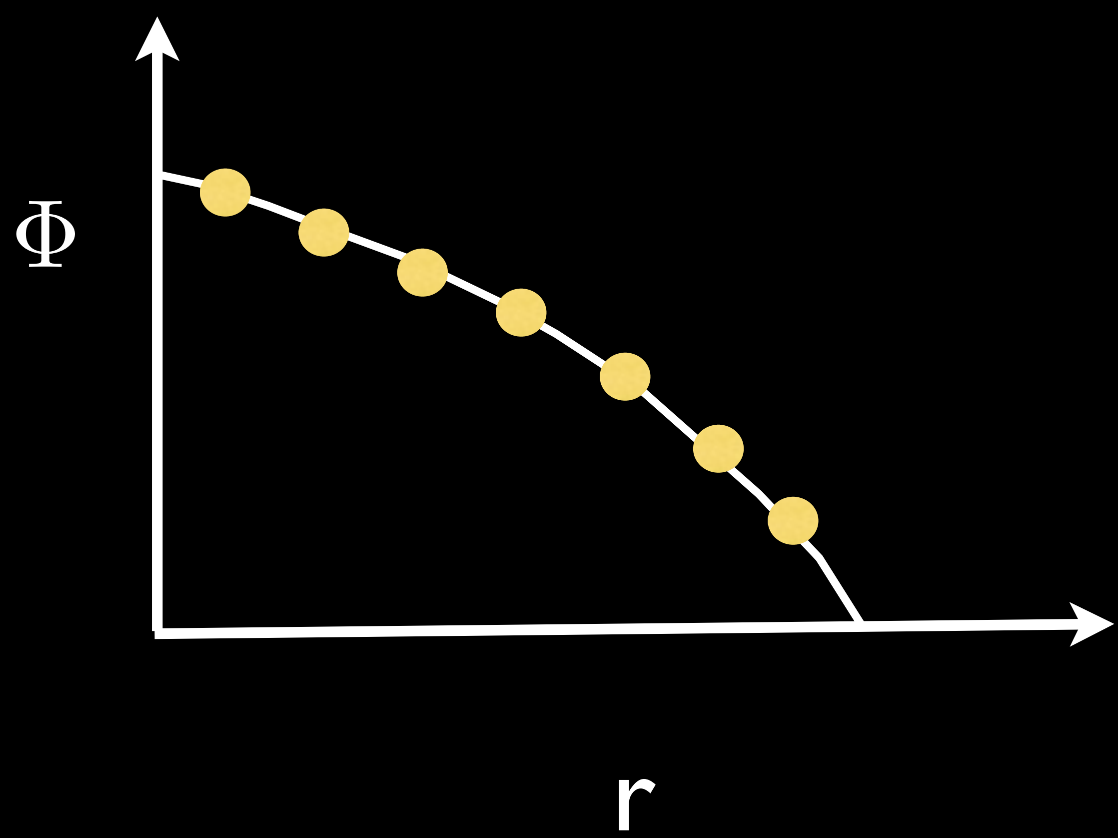
Brown+05

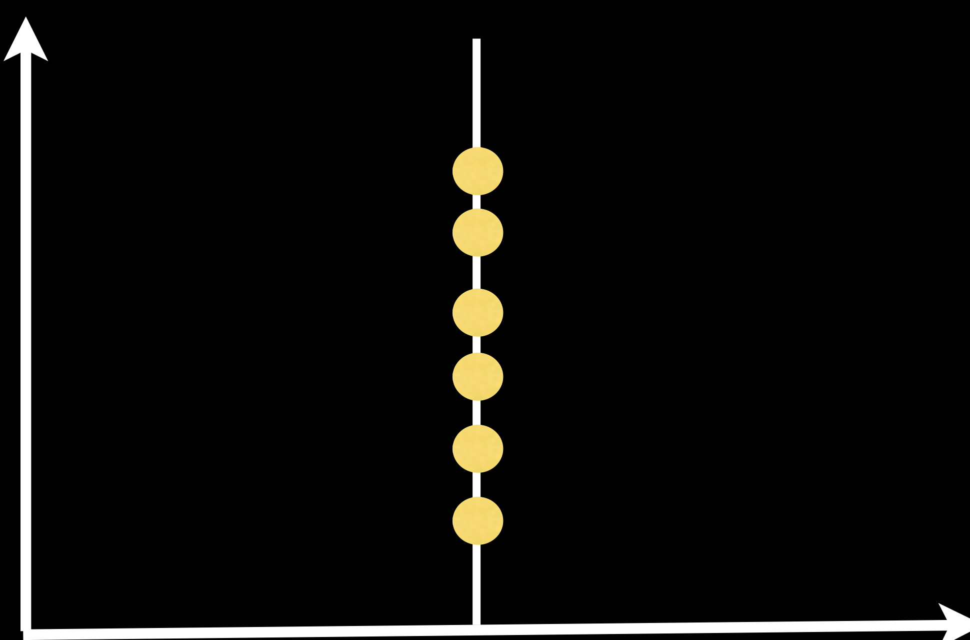
THE IDEA

Peñarrubia, Koposov & Walker (2012)

$$f(E) = \delta(E - E_0)$$

Delta function





$$E = \frac{1}{2}v^2 + \Phi(r)$$

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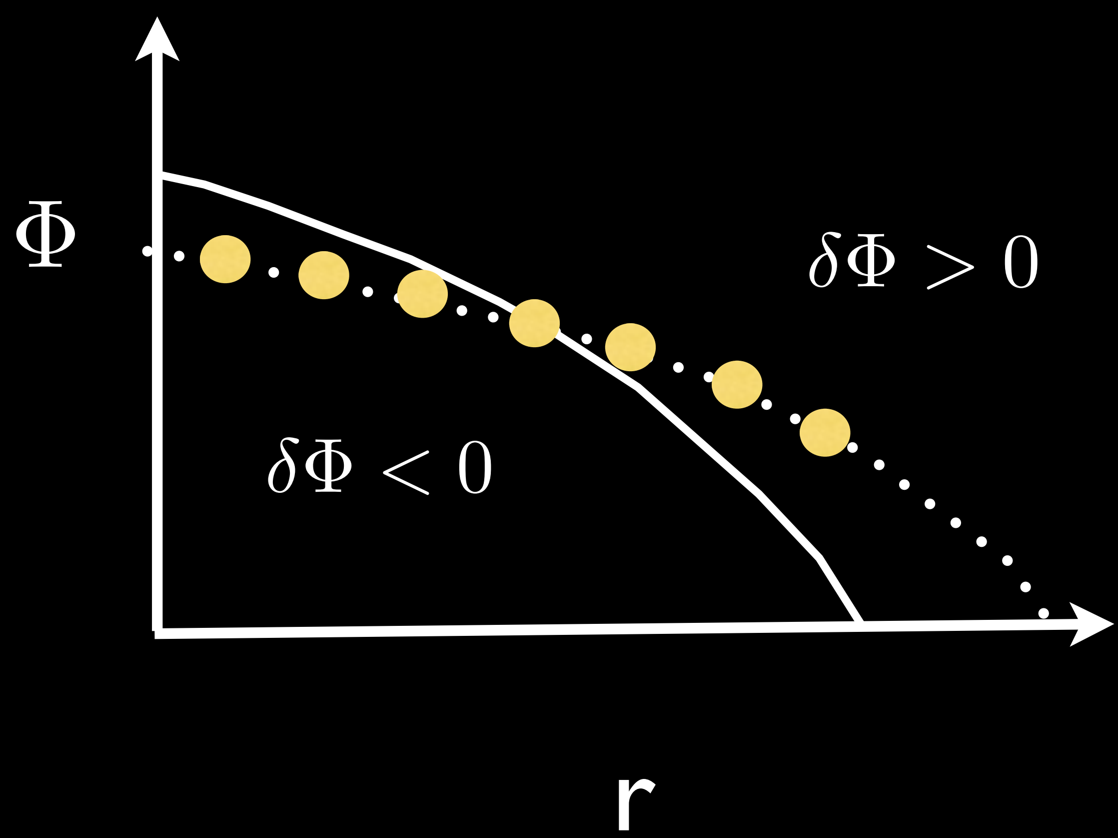
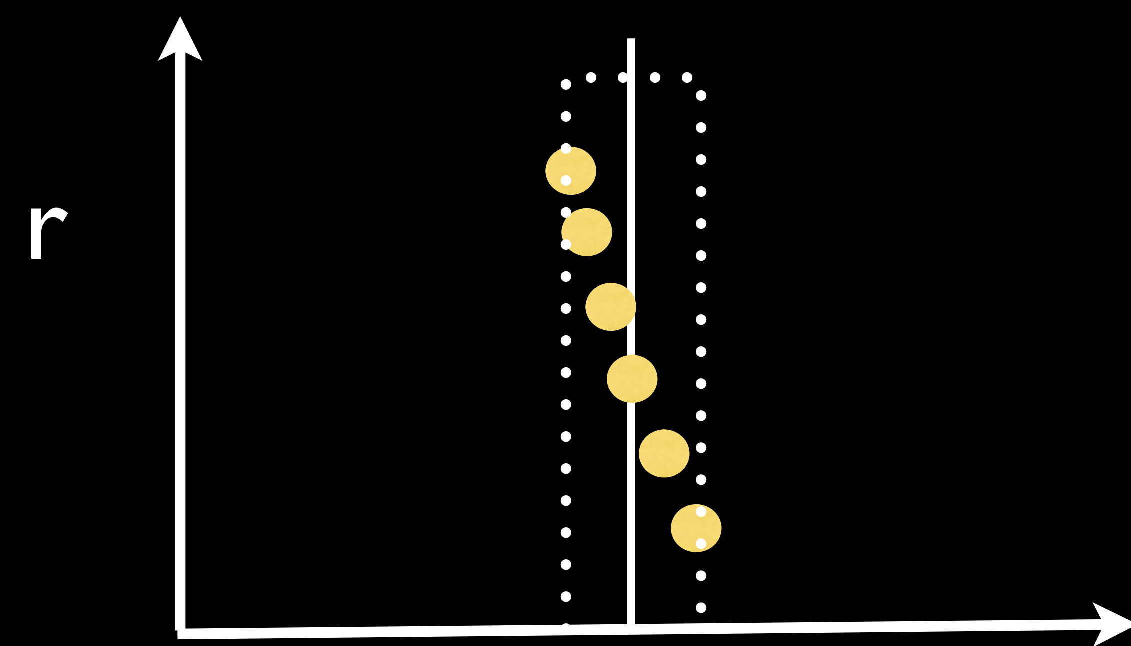
$$\sigma_E^2 = \sigma_\Phi^2 = \langle \delta\Phi^2 \rangle - \langle \delta\Phi \rangle^2$$





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$\delta\Phi(r)$ Bias arising from poor our knowledge of the potential

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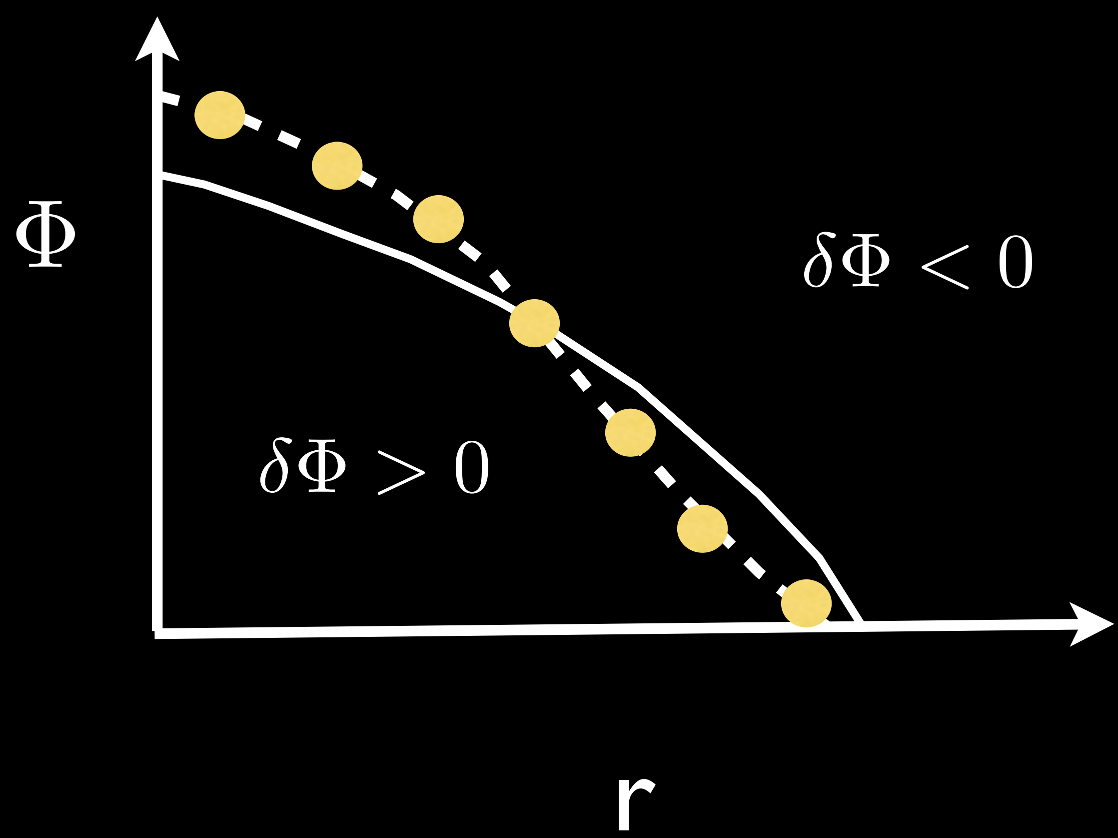

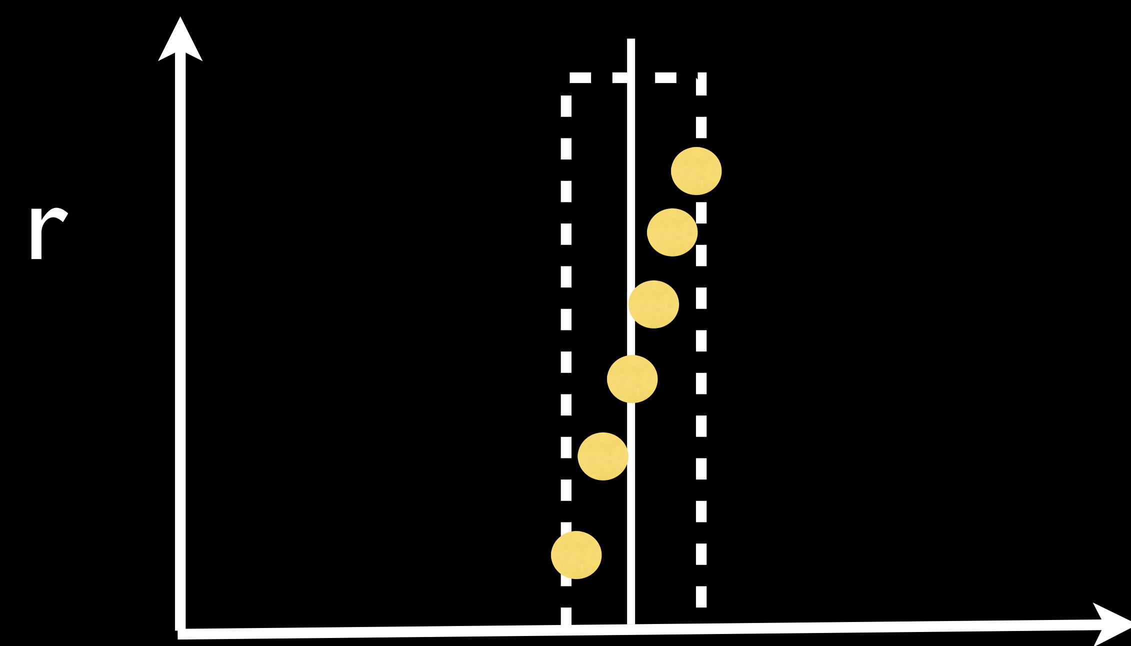

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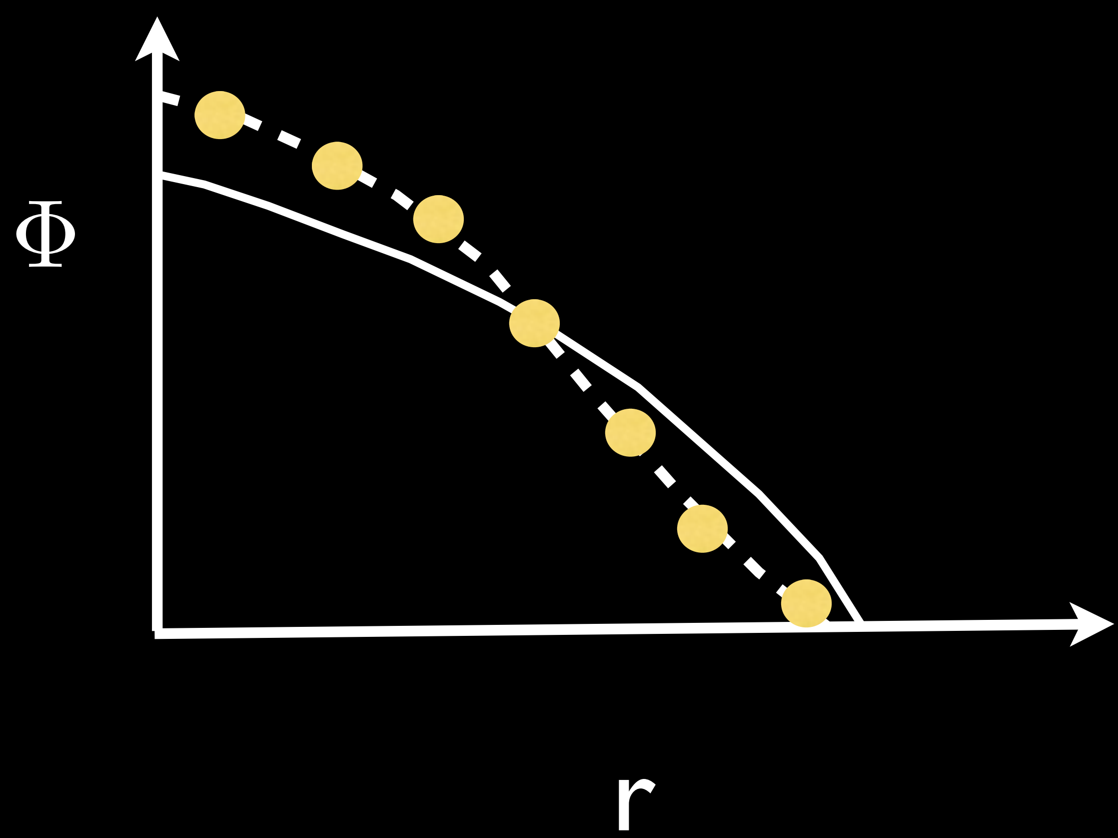
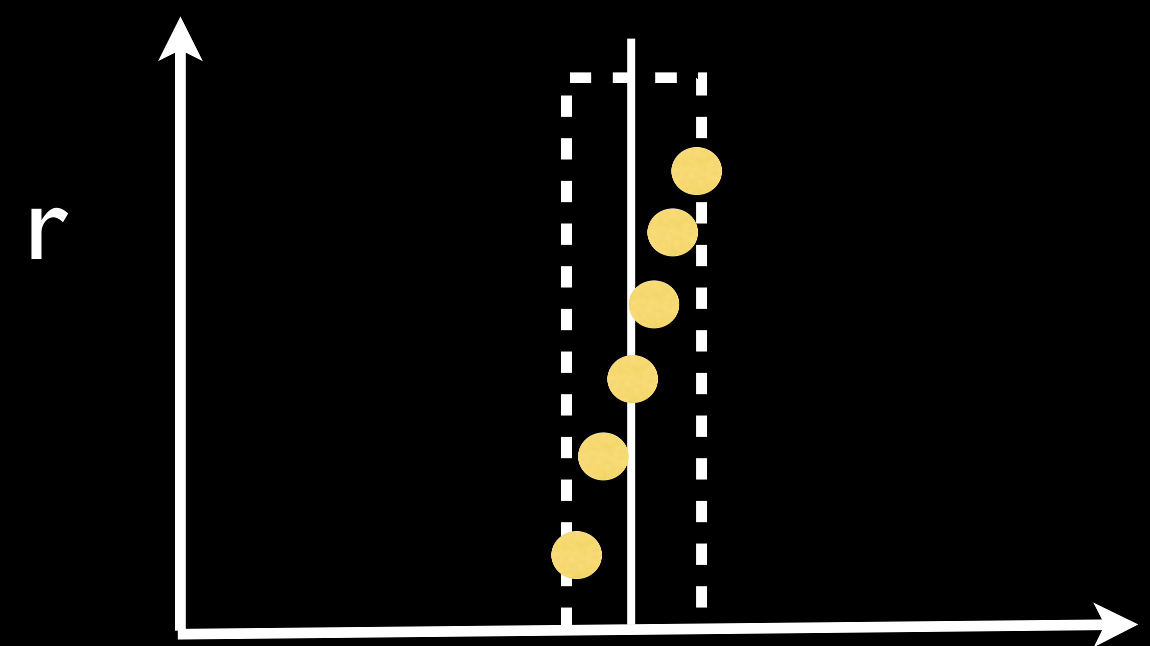
$$f(E) = \delta(E - E_0)$$

$$H \equiv -\int f(E) \ln f(E) dE = 0$$

$$\tilde{f}(E) \longrightarrow \Delta H > 0$$





$$E = \frac{1}{2}v^2 + \Phi(r)$$

“Biases in the calculus of orbital energy yields and ***increase*** *in the entropy of the energy distribution”*

Entropy

Theorem:

"The entropy measured for a stellar system with **separable** energy distribution *increases* under the presence of biases in the theoretical modelling of the host's gravity"

$$\varepsilon = -E + \Phi_\infty$$ Relative energy

$$\tilde{\varepsilon}(\mathbf{r}) = \varepsilon(\mathbf{r}) + \delta\Phi(\mathbf{r})$$ **Energy Bias**

$$\tilde{f}(\varepsilon, \mathbf{r}) = f[\varepsilon - \delta\Phi(\mathbf{r}), \mathbf{r}] = f[\varepsilon - \delta\Phi(\mathbf{r})]g(\mathbf{r})$$ Separability condition

Measured energy distribution:

$$\tilde{f}(\varepsilon) = \int f(\varepsilon - \delta\Phi(\mathbf{r}))g(\mathbf{r})d^3\mathbf{r} \approx$$

$$f(\varepsilon)\int \left[1 - \delta\Phi(\mathbf{r})\frac{f'(\varepsilon)}{f(\varepsilon)} + \frac{\delta\Phi^2(\mathbf{r})}{2}\frac{f''(\varepsilon)}{f(\varepsilon)}\right]g(\mathbf{r})d^3\mathbf{r} =$$

$$f(\varepsilon)\left[1 - \langle\delta\Phi\rangle\frac{f'(\varepsilon)}{f(\varepsilon)} + \frac{\langle\delta\Phi^2\rangle}{2}\frac{f''(\varepsilon)}{f(\varepsilon)}\right].$$

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Measured Entropy

$$\tilde{H} = -\int d\varepsilon \tilde{f}(\varepsilon)\ln[\tilde{f}(\varepsilon)] =$$

$$H + \langle\delta\Phi\rangle\int d\varepsilon f'(\varepsilon)[1 + \ln f(\varepsilon)]$$

$$-\frac{\langle\delta\Phi\rangle^2}{2}\int d\varepsilon f(\varepsilon)\left[\frac{f'(\varepsilon)}{f(\varepsilon)}\right]^2 - \frac{\langle\delta\Phi^2\rangle}{2}\int d\varepsilon f''(\varepsilon)[1 + \ln f(\varepsilon)].$$

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$$1)\int d\varepsilon\, f'(1+\ln f) = \left(f\ln f\right)_0^{\Phi_\infty} = 0,$$

$$2)\int d\varepsilon\, f''(1+\ln f) = -\int d\varepsilon\, f\left[\frac{f'}{f}\right]^2.$$

$$\tilde{H} = H + \frac{\langle\delta\Phi^2\rangle - \langle\delta\Phi\rangle^2}{2}\int d\varepsilon\, f(\varepsilon)\left[\frac{f'(\varepsilon)}{f(\varepsilon)}\right]^2 \equiv H + \frac{\sigma_\Phi^2}{2\sigma_\varepsilon^2} \quad \geq 0$$

Entropy

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- Entropy increases for $\delta\Phi = \delta\Phi(\mathbf{r}) \neq 0$
- Adding a constant value to the potential does not yield an increase in entropy
- Changes in entropy will be stronger for "cold" distributions

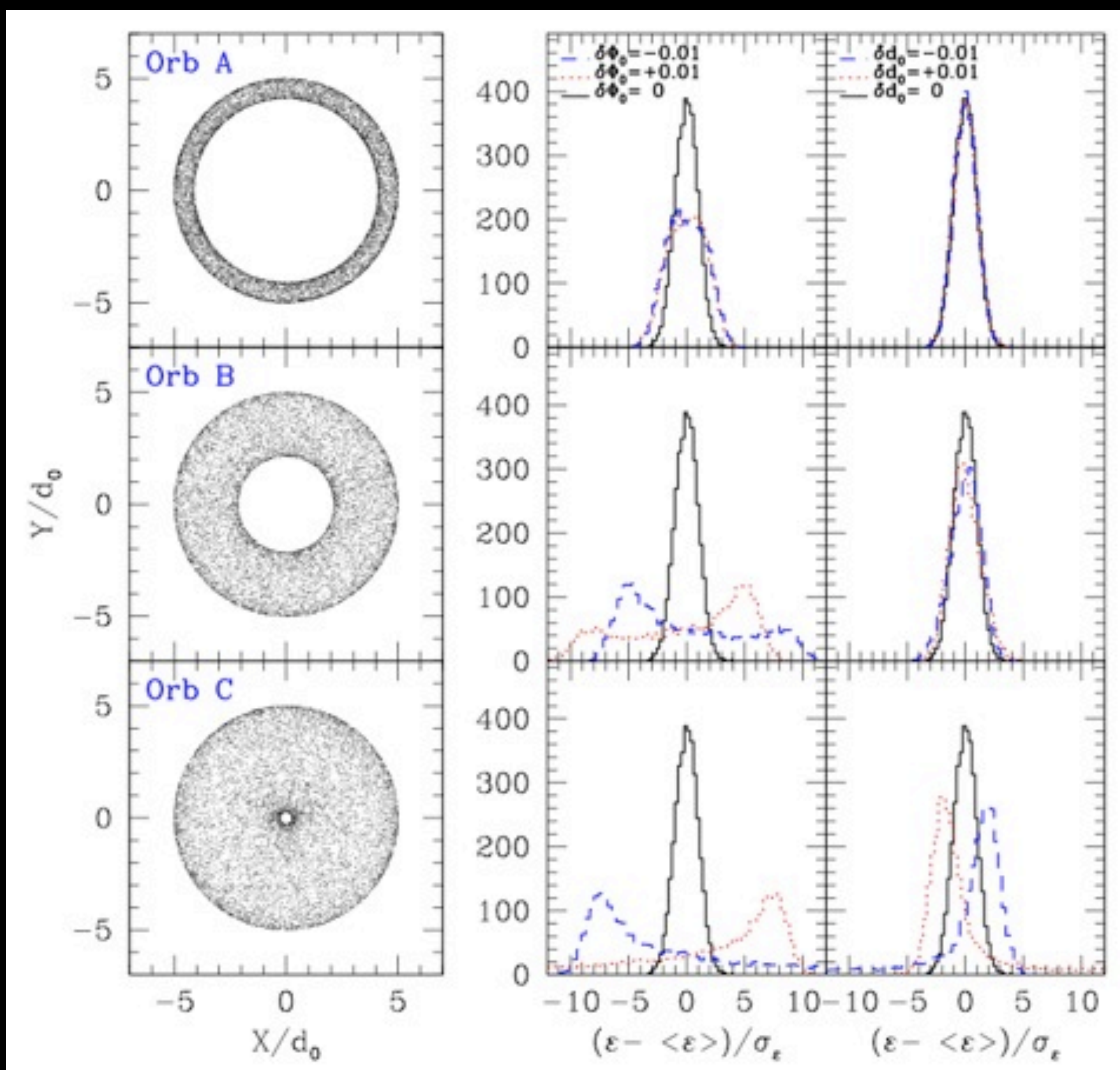
Tests

$$f(\varepsilon) = 1/\sqrt{2\pi\sigma_\varepsilon^2}\exp[-(\varepsilon-\varepsilon_{\rm orb})^2/(2\sigma_\varepsilon^2)$$

Unbiased (true) energy distribution

$$\Phi(r) = \Phi_0 \ln(d_0^2 + r^2)$$

Unbiased (true) Potential



$$r_{\rm apo} = 5d_0$$

$$\sigma_\varepsilon = 10^{-3}\Phi_0$$

$$H_{\rm Gauss} = 1/2[\ln(2\pi\sigma_\varepsilon^2) + 1]$$

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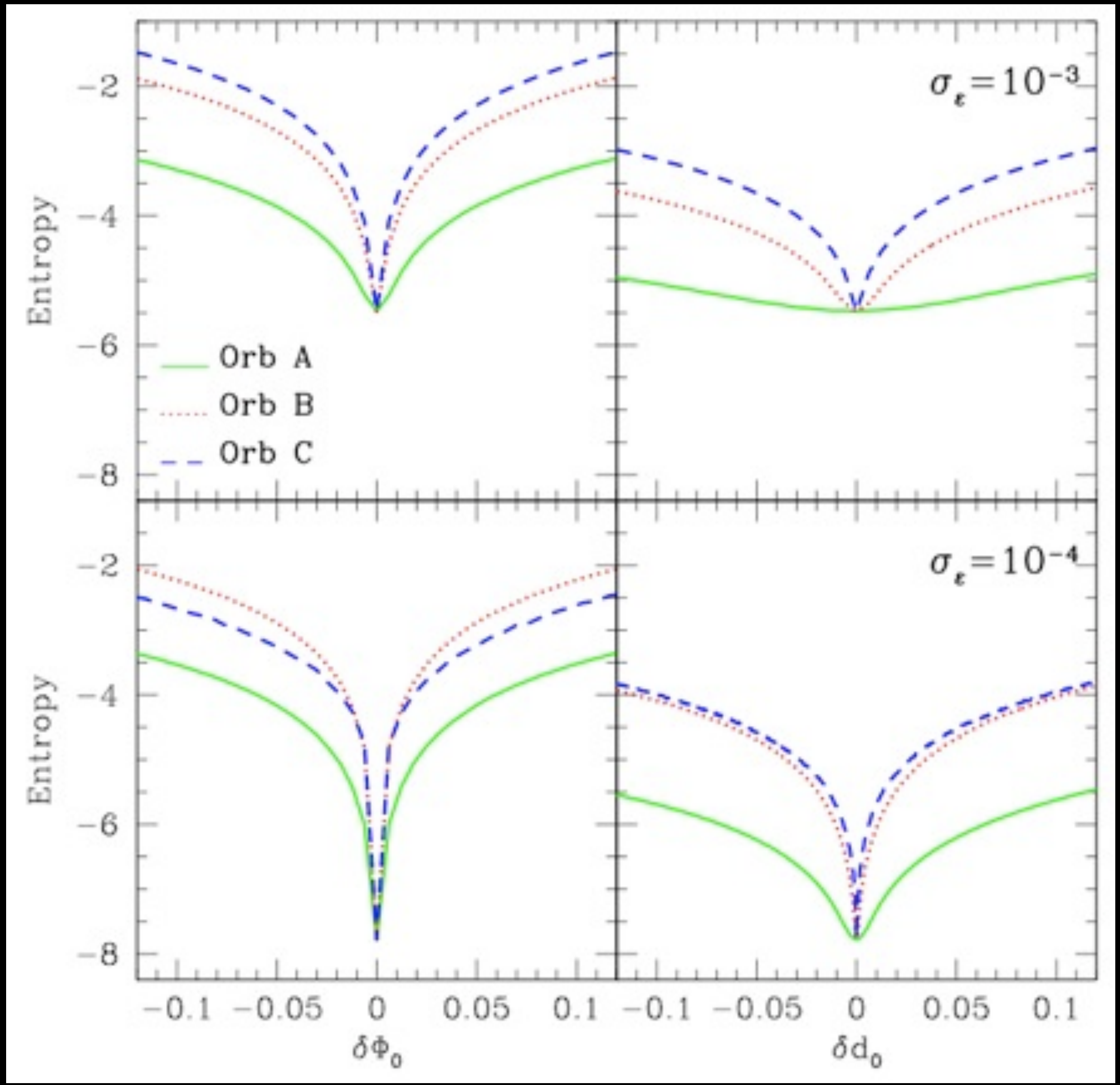

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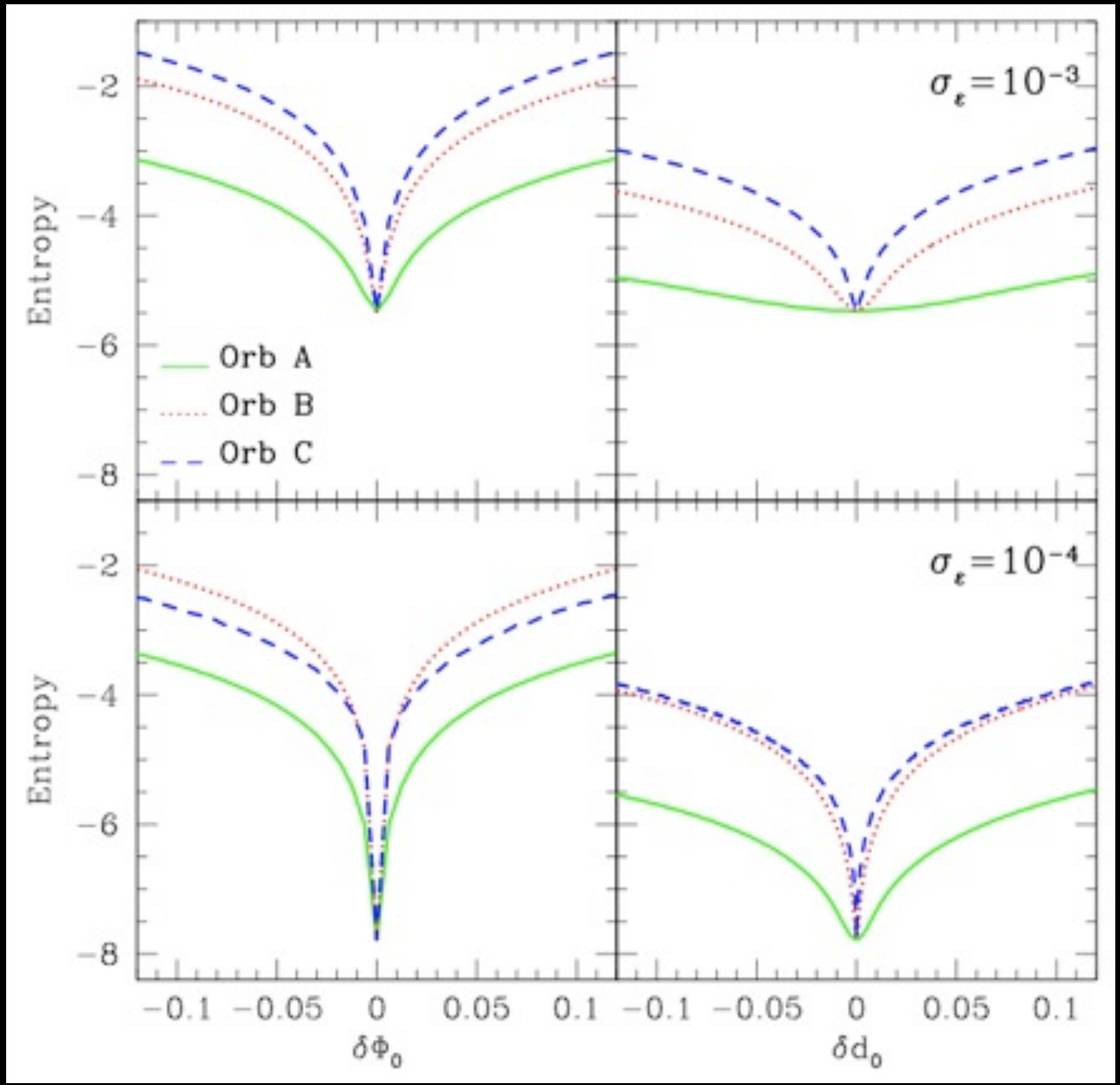

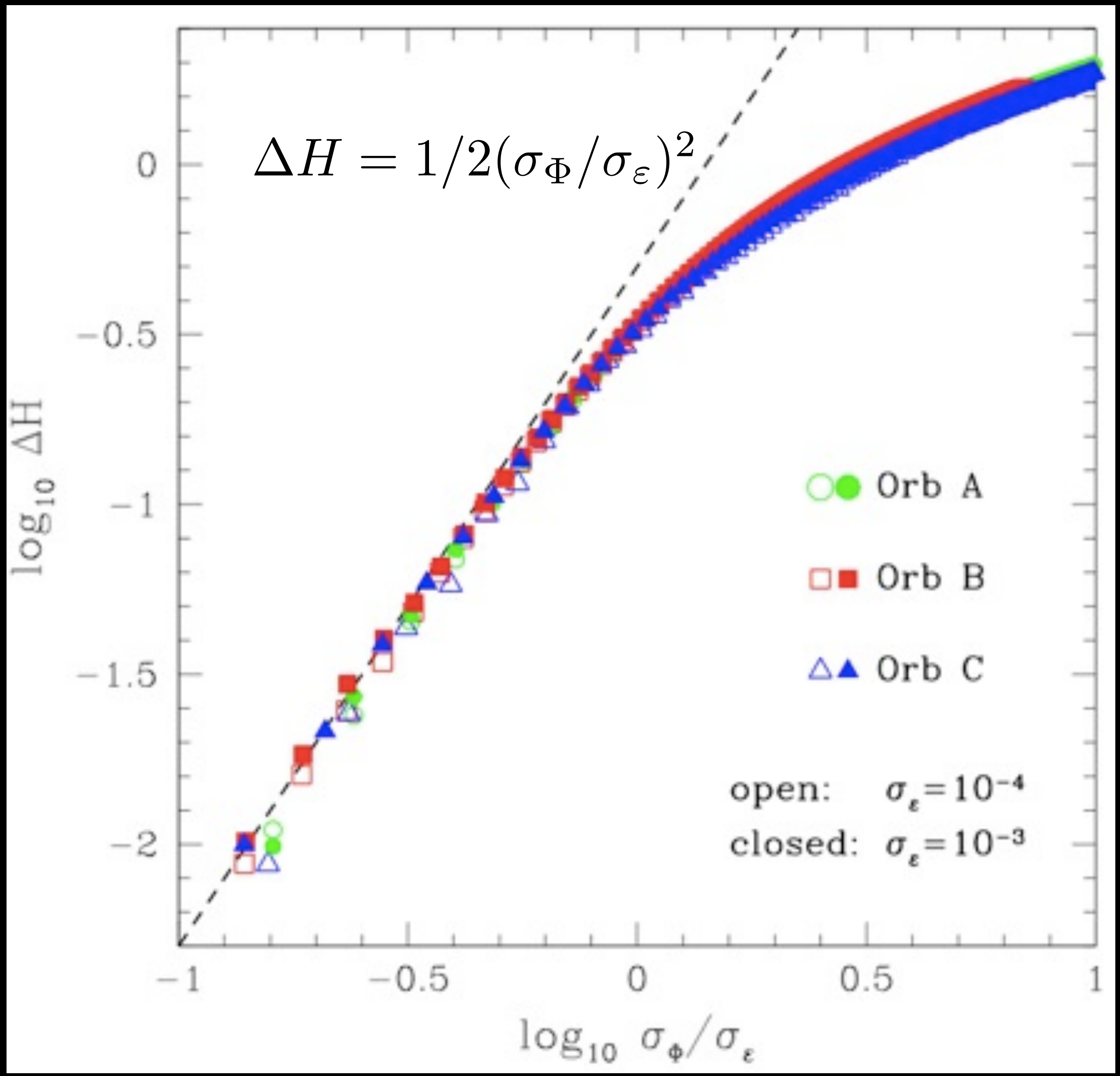

Energy biases

1. Potential parameters
2. Functional form of the potential
3. Gravity model

$$\tilde{\Phi}(r) = 2\Phi_0 \left[y + \frac{y^3}{3} + \frac{y^5}{5} + ... + \sum_{k=0}^{(N-1)/2} y^{2k+1}/(2k+1) \right] + \Phi_0 \ln d_0^2$$

$$\lim_{N\to\infty} \tilde{\Phi} = \Phi_0 \ln(r^2 + d_0^2) = \Phi$$

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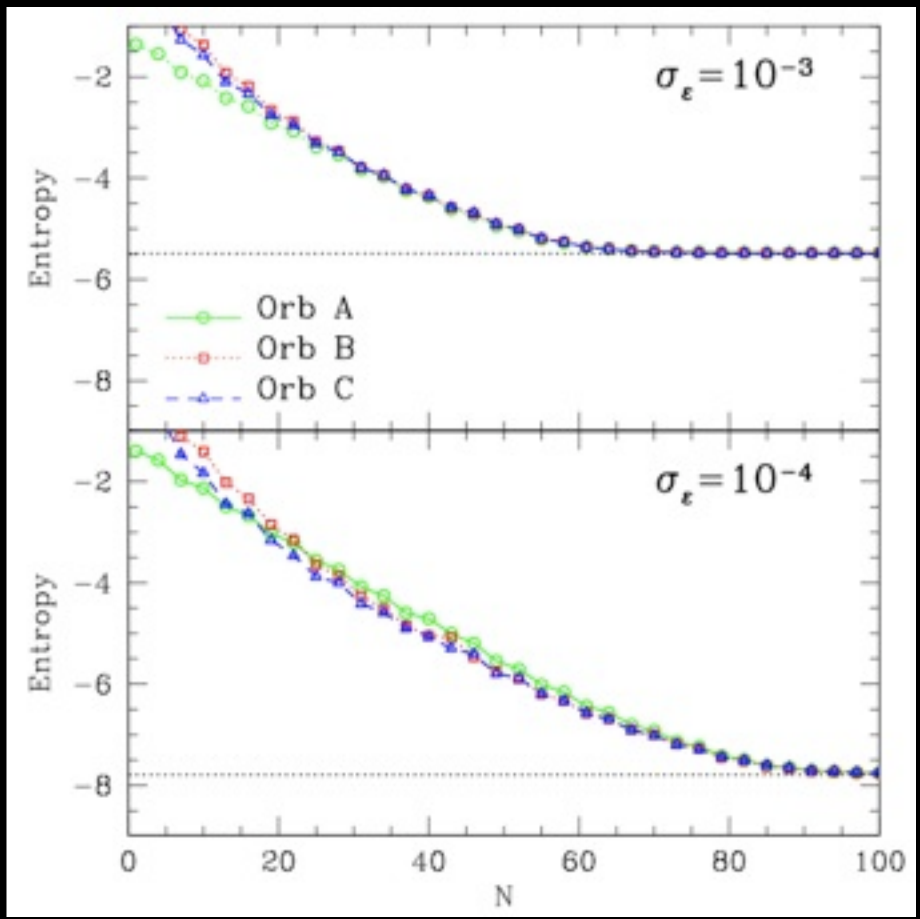

Entropy can be used to distinguish between different potential parametrizations

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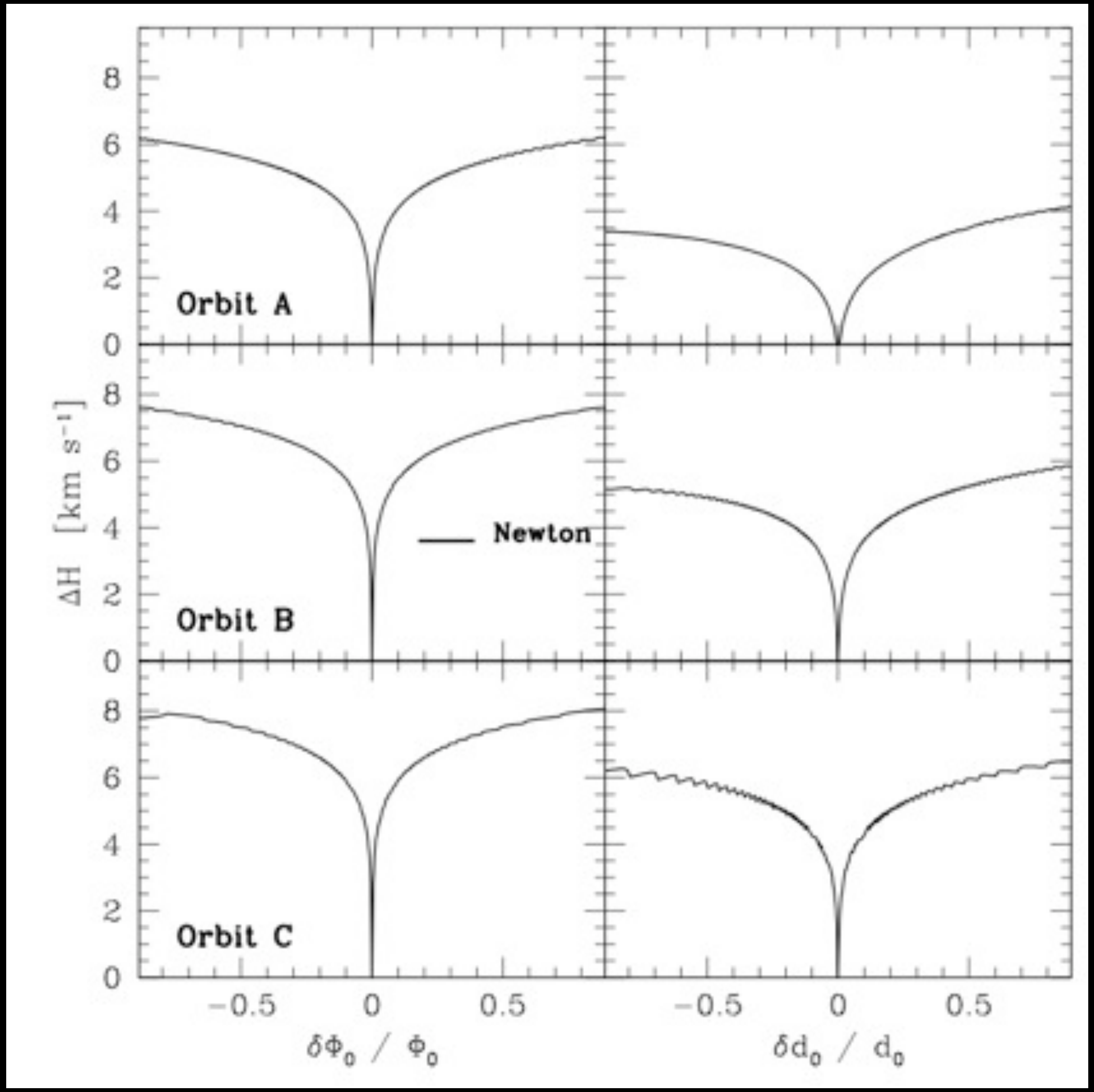

Example 1: Dirac's cosmology

$$\frac{Gm_p m_e}{e^2} \simeq 10^{-39} \simeq \frac{e^2}{m_e c^3 t};$$

$$E_D = H_0^2 t^2 \left[\frac{1}{2} \left(\frac{d\mathbf{r}}{dt} \right)^2 + \frac{G}{G_0} \Phi(\mathbf{r}) - \left(\frac{d\mathbf{r}}{dt} \cdot \frac{\mathbf{r}}{t} \right) \right] + \frac{1}{2} H_0^2 \mathbf{r}^2;$$

Lynden-Bell (1982)

at $t=H_0^{-1}$

$$\delta\Phi_D = \pm[-H_0(d\mathbf{r}/dt \cdot \mathbf{r}) + 1/2 H_0^2 \mathbf{r}^2].$$

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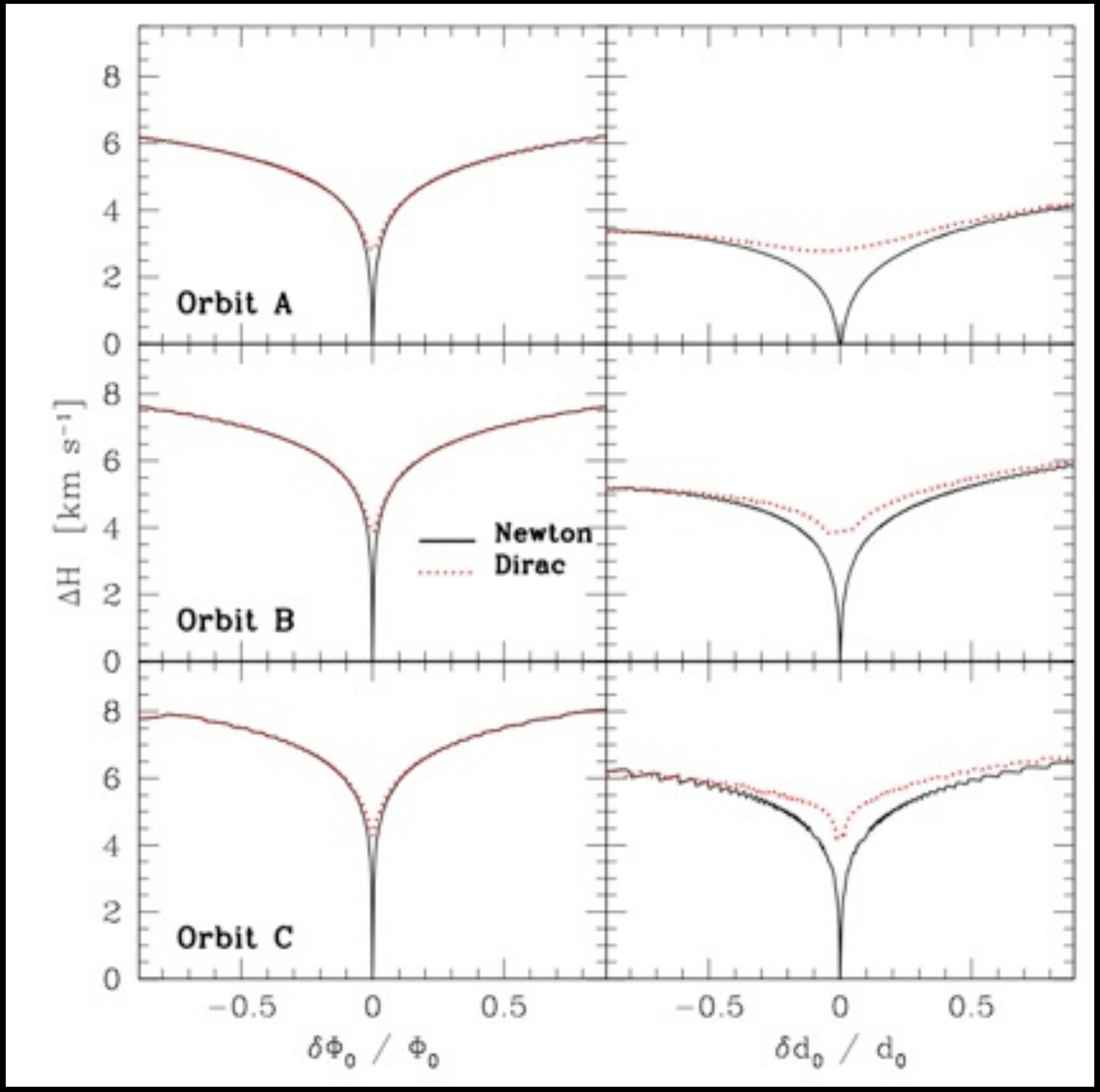

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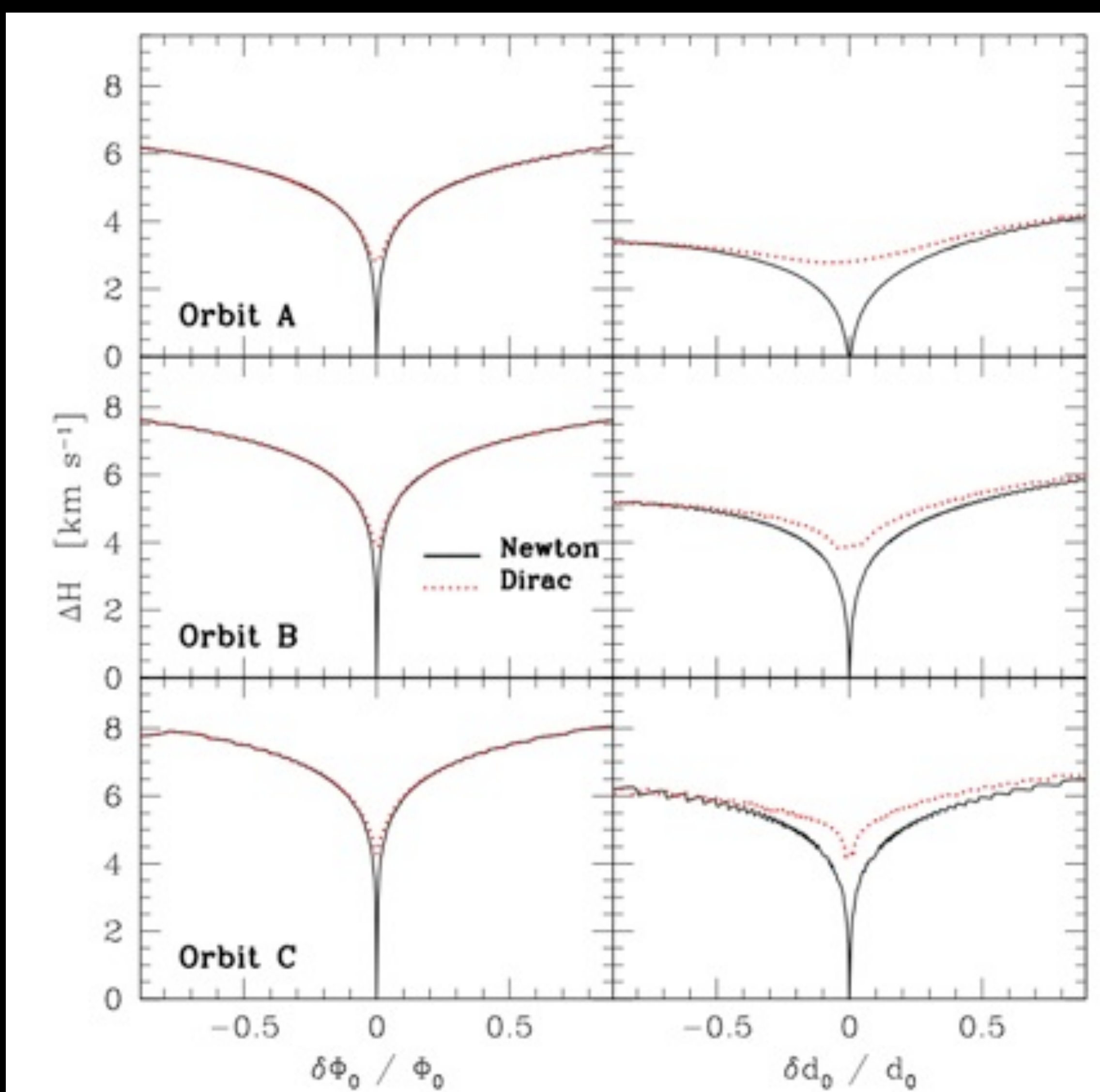

Example 2: QMOND

$$\mathbf{g}_M = \mathbf{g}_N \nu(r) \equiv \mathbf{g}_N \left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{a_0}{g_N}} \right),$$

$$g_N = -GM(< r)/r^2,$$

$$\Phi_M(r) = \int_r^{\infty} g_M(r') \mathrm{r}';$$

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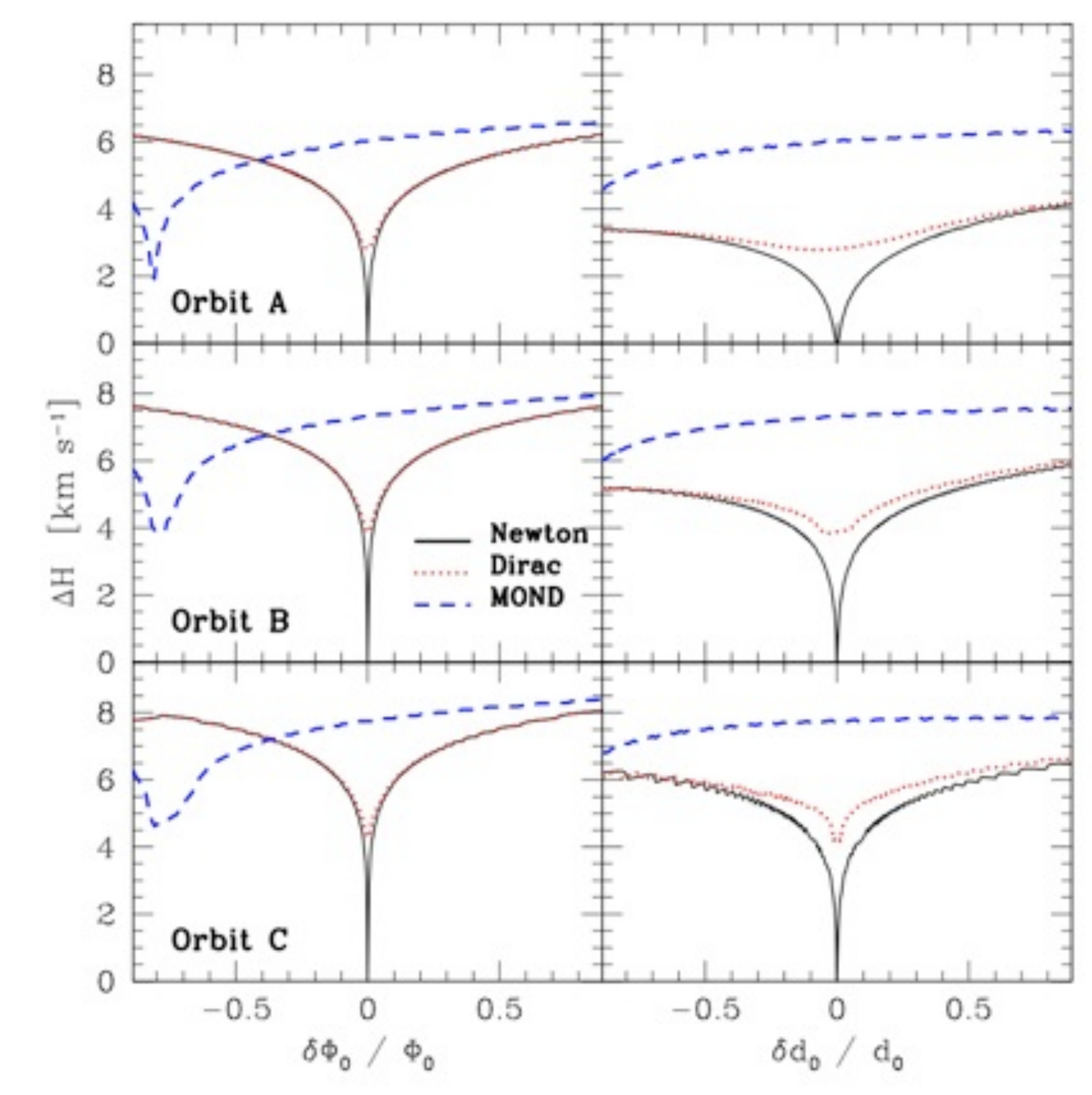

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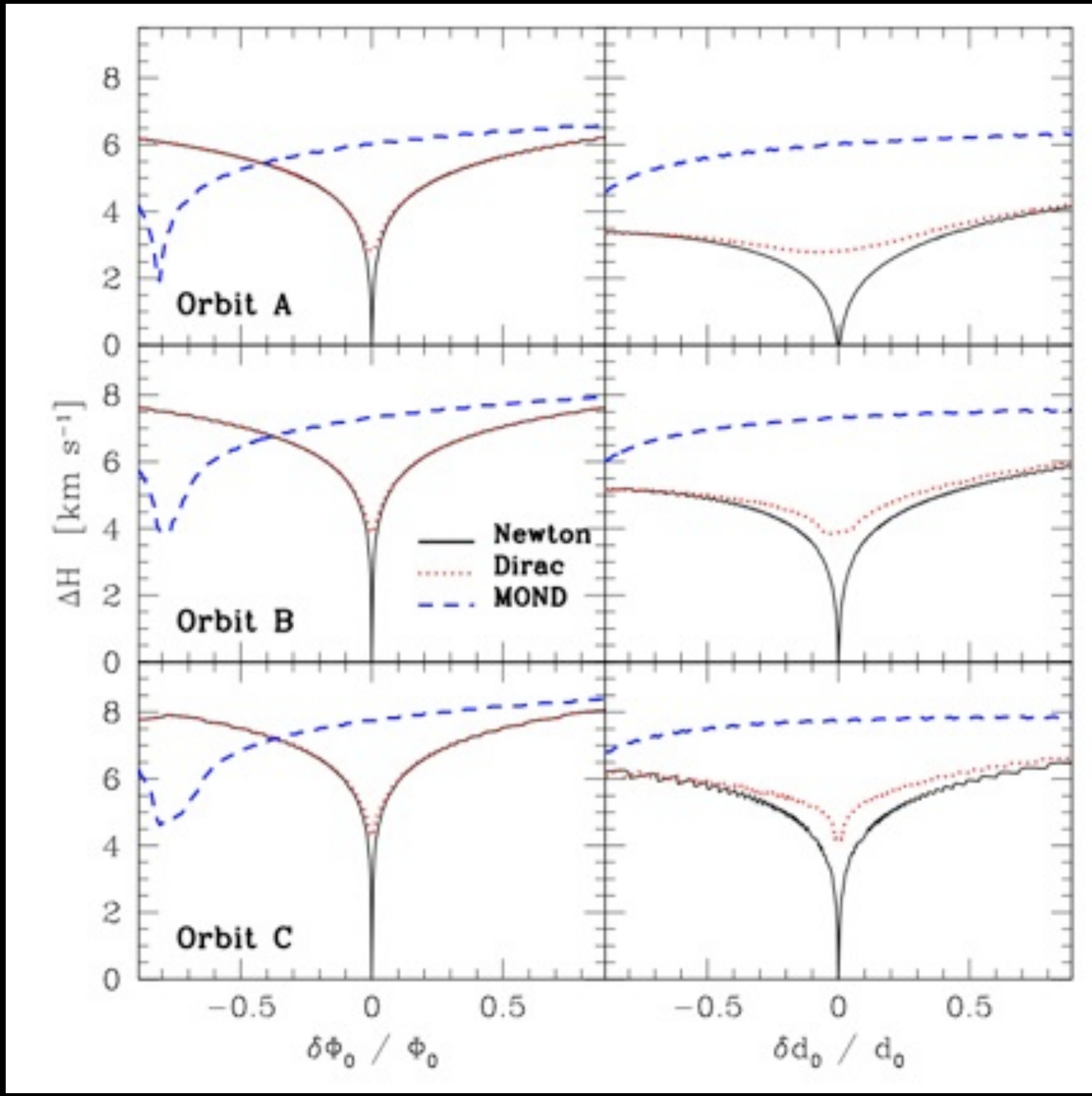

Example 3: f(R) gravity theories

$$\mathcal{A} = \int d^4x\sqrt{-g}[f(R) + \mathcal{L}_m];$$

$$f(R) = f_0 R^n$$ Ricci curvature

ΛCDM: $f(R) = R + 2\Lambda$

Cappozziello et al (2007)

$$\Phi_R = 1/2(\Phi_N + \Phi_c)$$

$$\Phi_c(r) = -4\pi G\left[\frac{1}{r}\int_0^r dr'\rho(r')r'^2\left(\frac{r}{r_c}\right)^\beta + \int_r^\infty dr'\rho(r')r'\left(\frac{r}{r_c}\right)^\beta\right].$$

$\beta = 0$ Newton

$\beta = 0.82$ Fit rotation curves with NO DM

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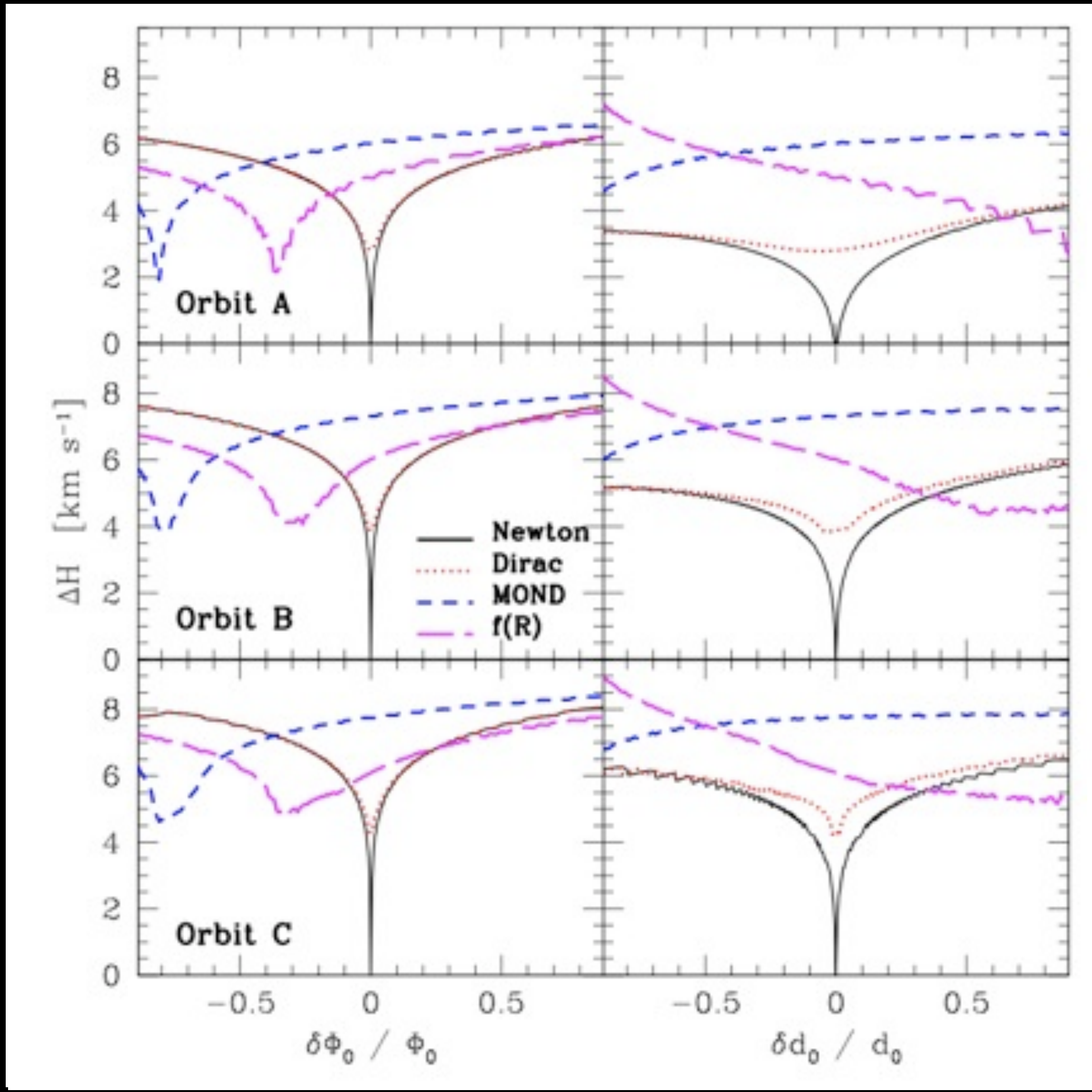

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The Minimum Entropy Method

1. Phase-space catalogue: $\{X,Y,Z,V_x,V_y,V_z\}_i$; $i=1,2,...,N_*$
2. *Identify clumps in integrals-of-motion space* (Luis' talk!)
3. Calculate $E_i=1/2(V_x^2+V_y^2+V_z^2)_i+\Phi(X_i,Y_i,Z_i)$
4. Calculate f(E), H
5. Look for Φ that minimizes H
6. Repeat for alternative gravity theories

Summary

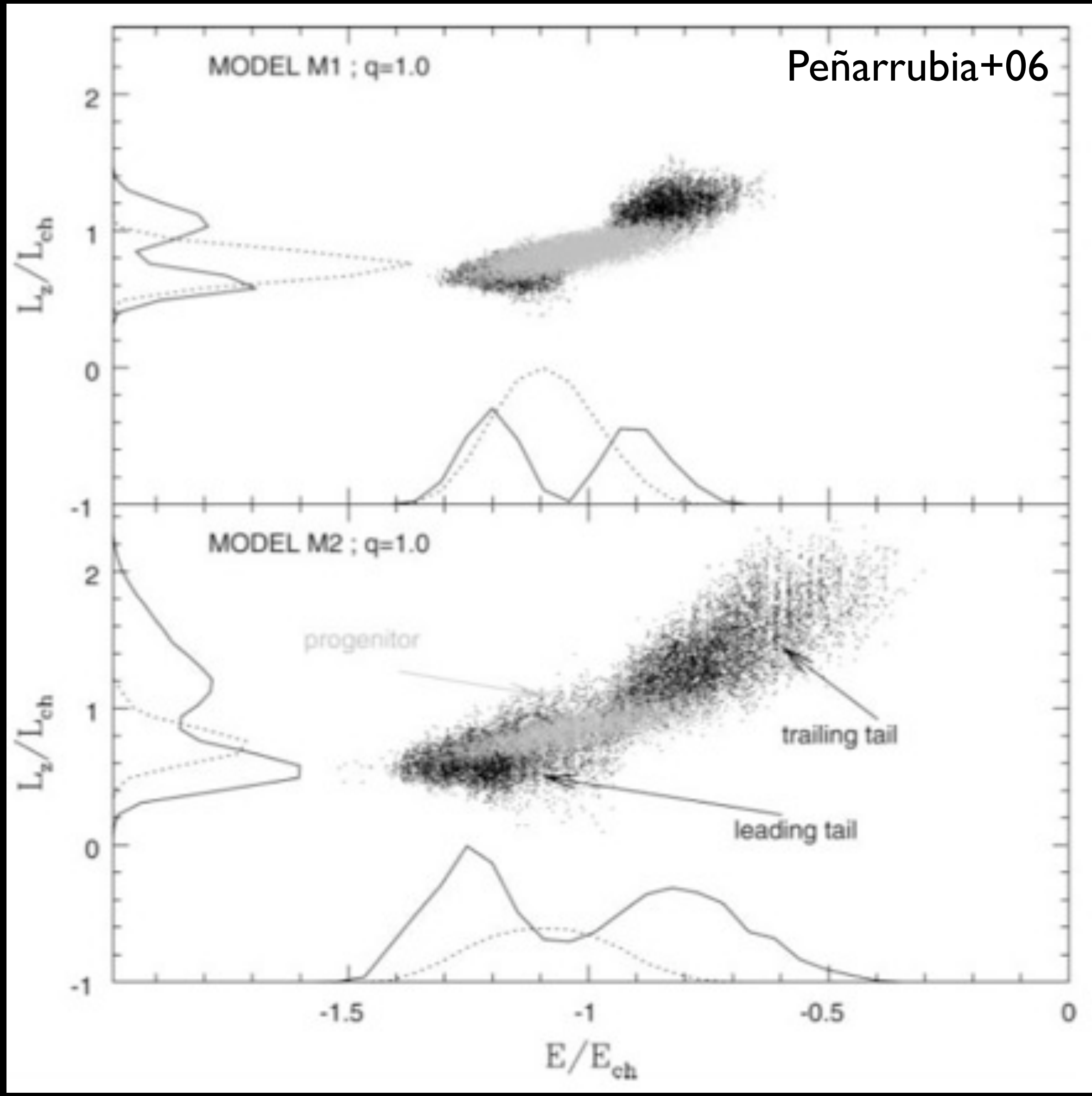
- “The true Milky Way potential is that that minimizes the entropy measured for stellar systems with separable energy distributions”
- Best targets: Tidal debris of satellites/clusters with low dynamical masses
- Future work: Gaia errors? MW background?

Tidal debris

Theorem:

"The entropy measured for stellar systems with separable energy distributions increases under the presence of biases in the theoretical modelling of the host's gravity"

Is the energy distribution of tidal debris separable?



Kullback-Leiblar (or KL) divergence

$$D_i = \int f_i(\varepsilon) \ln \left[\frac{f_i(\varepsilon)}{f(\varepsilon))} \right] d\varepsilon \equiv -H_i + H_{c,i};$$

where

$$H_{c,i} = -\int f_i(\varepsilon) \ln f(\varepsilon) d\varepsilon$$ Crossed entropy

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$$H = -\int f(\varepsilon) \ln f(\varepsilon) d\varepsilon = -\alpha \int f_l(\varepsilon) \ln f(\varepsilon) d\varepsilon - (1-\alpha) \int f_t(\varepsilon) \ln f(\varepsilon) d\varepsilon$$

$$\equiv \alpha H_l + (1-\alpha) H_t + \alpha D_l + (1-\alpha) D_t \equiv \langle H \rangle_{l,t} + \langle D \rangle_{l,t};$$

Distributions are separable if D_i=0

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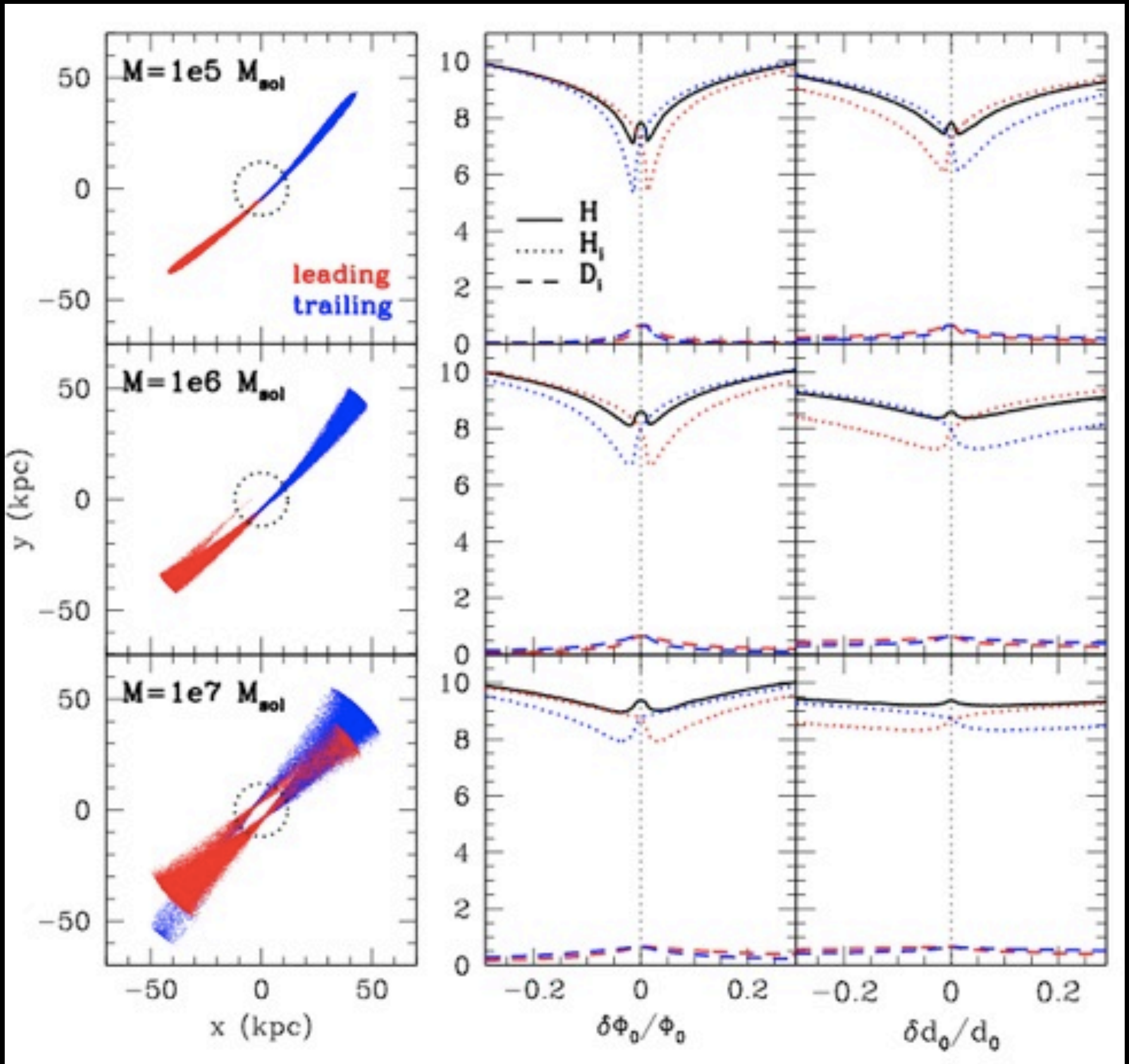


$$H = \langle H\rangle_{l,t} + \langle D\rangle_{l,t};$$

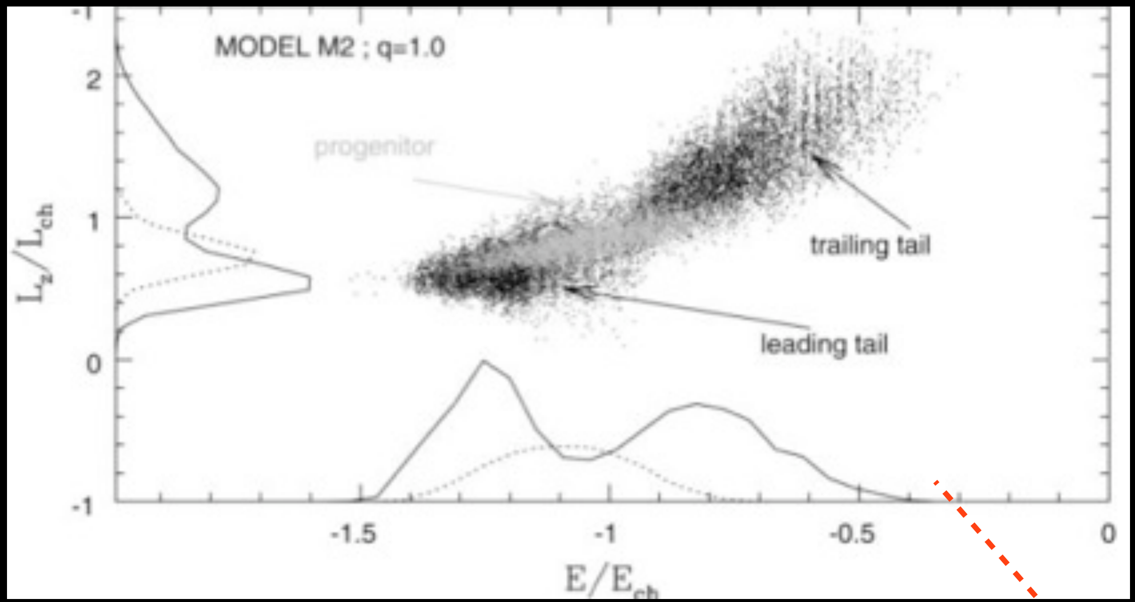
↑ minimum if δΦ=0

↑ maximum if δΦ=0

maximum in H δΦ~0 $\langle H\rangle'_l = \langle D\rangle'_l = 0$

minimum in H δΦ~0 $\langle H\rangle'_l = -\langle D\rangle'_l$

Peñarrubia et al. (2006)
“Modeling Tidal Streams in Evolving Dark Matter Halos”



$t=t_{now}$